OP&P Product Research –
Utrecht, The Netherlands
May 23rd, 2011

Searching for a Single Grain of Sand: Finding Most Compatible Combinations of Ingredients, Flavors or Components

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Optimal Combinations

- Many practical problems involve optimizing combinations

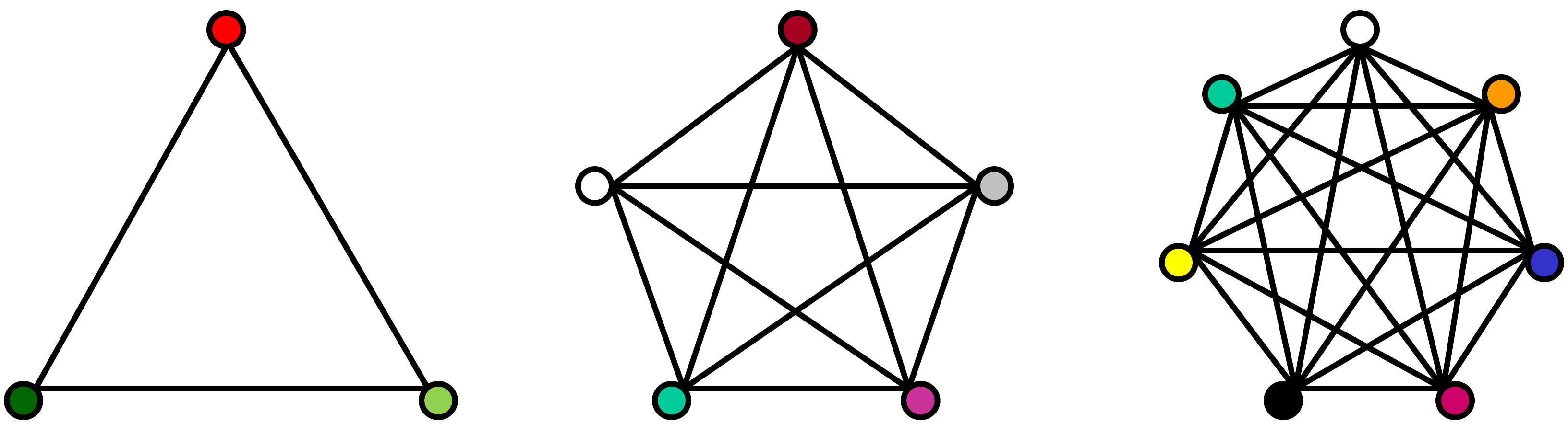
  - Ingredient combinations
    - Pizzas 
    - Juices 
    - Salads 

  - Flavor combinations
    - Potato chips 
    - Sauces 
    - Candy bars 

  - Component or feature combinations
    - Boxed lunches 
    - Meals ready to eat 
    - Automobiles 
    - Political candidates 
    - …

Problem with Combinations

- As number of possible choices for each item increases, number of possible combinations explodes



- Need to avoid consideration of all combinations

Example - Pizza Menu Optimization

Example from collaborative research with Michael Nestrud

(Food Science Department - Cornell University)

Example – Pizza Menu Optimization

- You work for a major pizza franchising business
- Goal: Create 5 pizzas with up to 5 toppings each
- Requirements:
  - 25 possible toppings
  - Each pizza should contain toppings that are as compatible as possible
  - Overall menu should appeal to as many consumers as possible
- Number of possible combinations?
  - There are 68406 possible pizzas with 5 or fewer toppings
  - Approximate number of possible menus: 12,500,000,000,000,000,000,000

Example – Pizza Menu Optimization

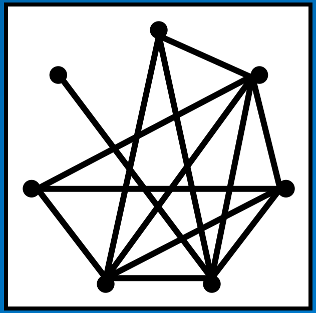
- 25 possible toppings:

Anchovy	Broccoli	Ground Sausage	Onion	Red Onion
Artichoke	Chicken	Ham	Pepperoni	Ricotta Cheese
Bacon	Eggplant	Italian Sausage	Pineapple	Roasted Garlic
Basil	Feta	Jalapeno	Prosciutto Ham	Spinach
Black Olive	Green Bell Pepper	Mushroom	Red Bell Pepper	Tomato

- Step 1: Find all optimal pizzas with 5 toppings or less
- Step 2: Find optimal menu made of 5 optimal pizzas
- Need to define optimal pizzas and optimal menu

Defining Optimal Pizzas

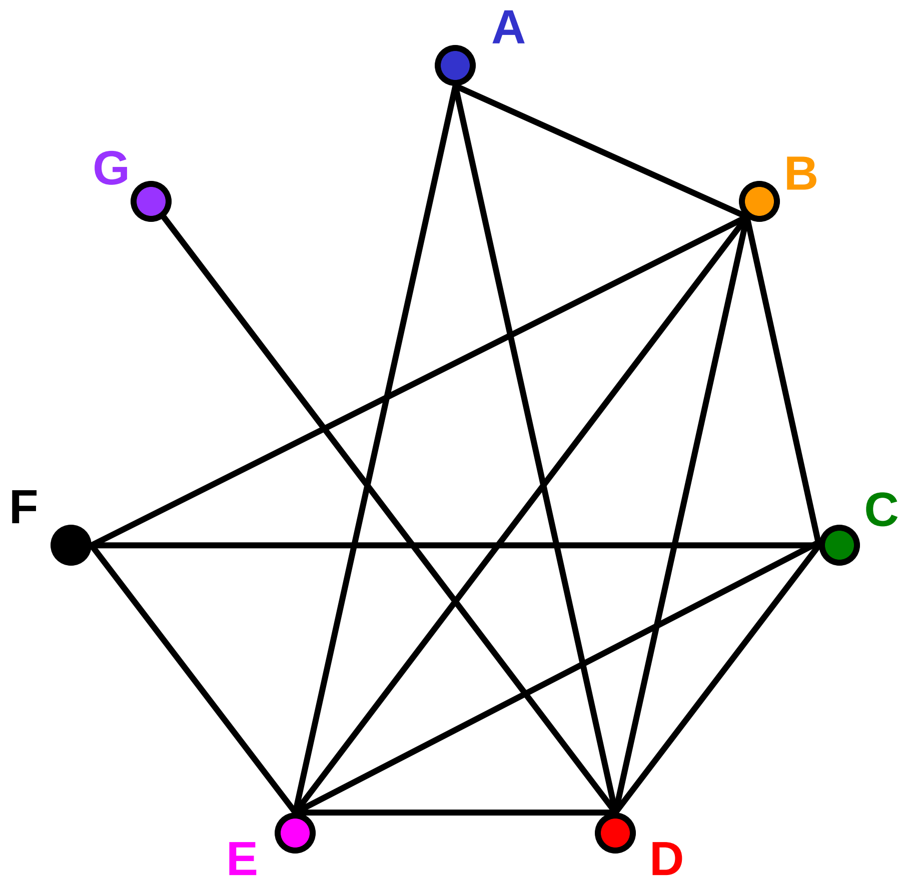
- There are 68,406 possible pizzas with 5 or fewer toppings
- How can we eliminate some of the combinations?
- Many combinations contain pairs that are not likely to be desirable
  - There are 1795 combinations that contain anchovy and ham
- If we can identify less desirable pairs, we can eliminate large numbers of combinations
- If we can identify desirable pairs, we can build optimal combinations
- Building optimal combinations from pairs uses graph theory



A Brief Introduction to Graph Theory

Graph Theory

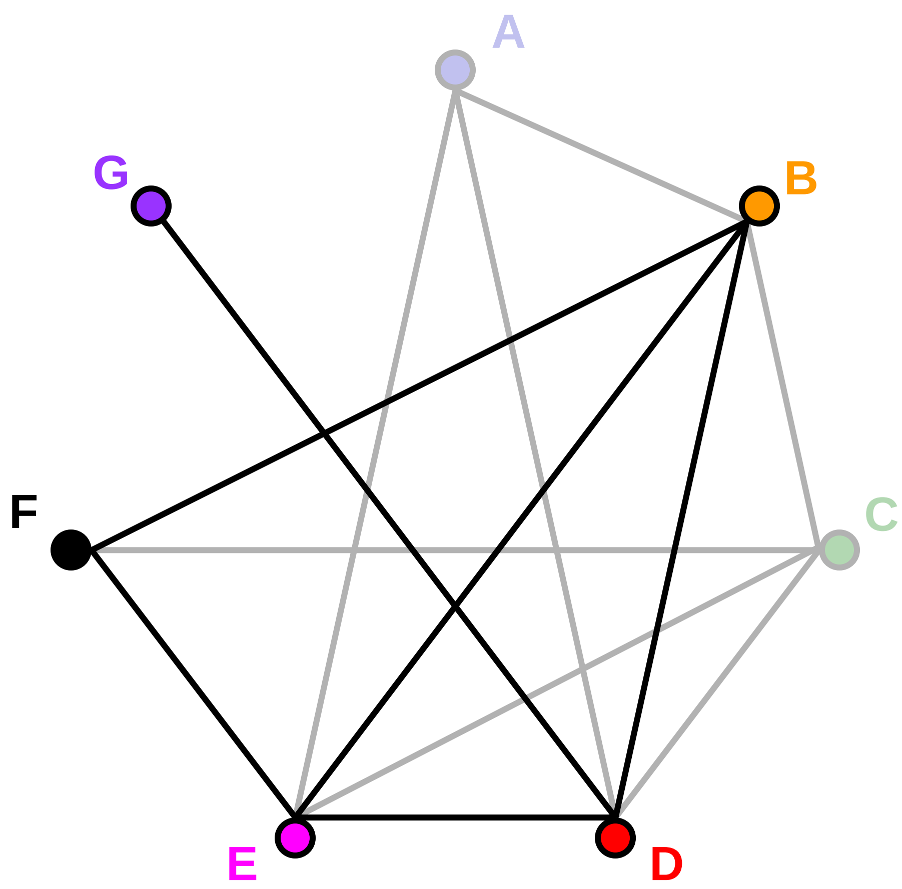
- A graph is a collection of objects together with connections



- Graph theory is the study of connections

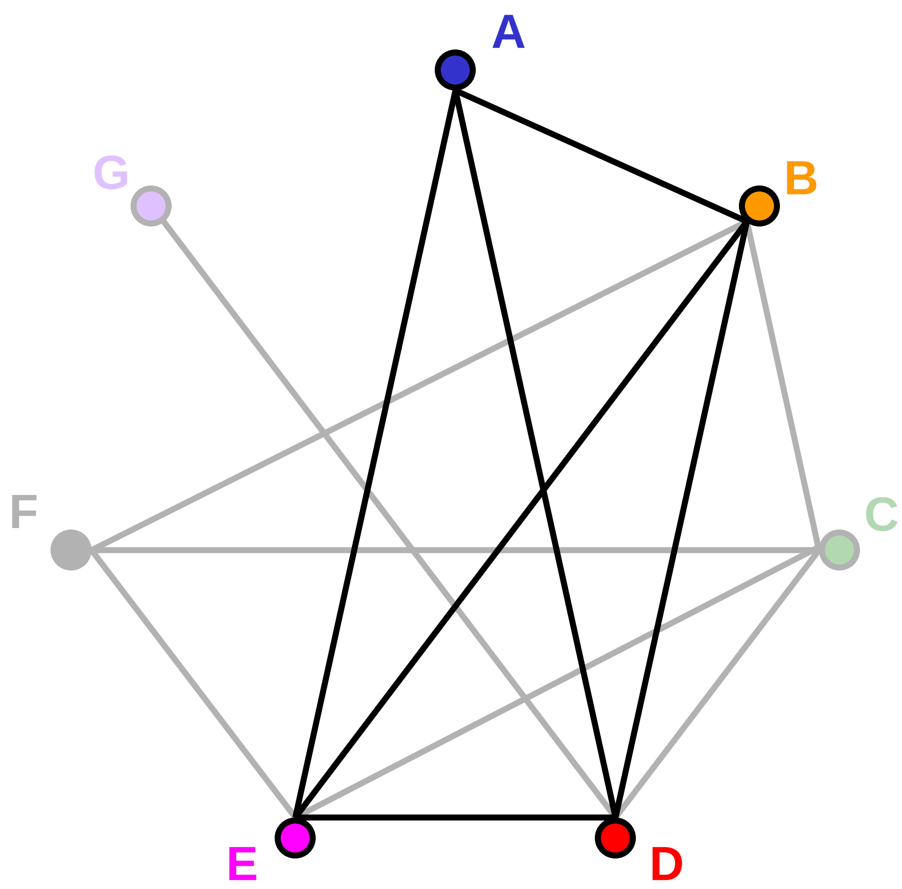
Subgraphs

- A **subgraph** is a collection of objects and connections within a graph



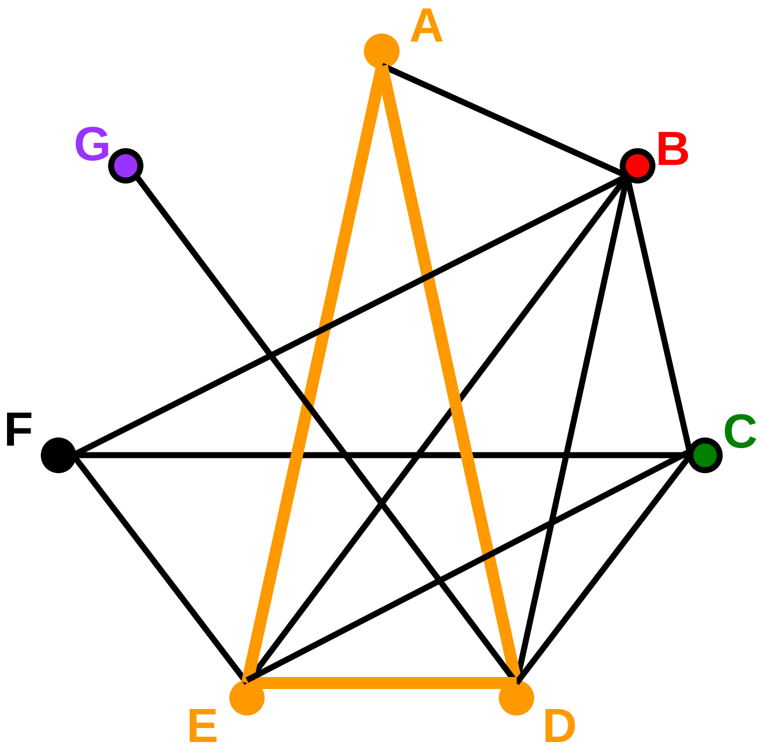
Cliques

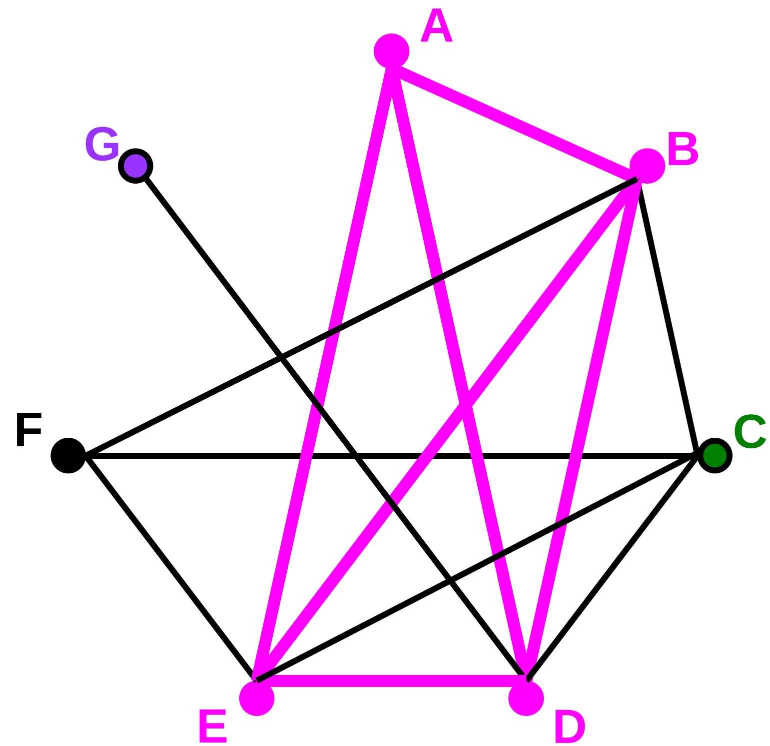
- A **clique** is a subgraph that is fully connected



Cliques *(cont.)*

- Cliques can be found within larger cliques

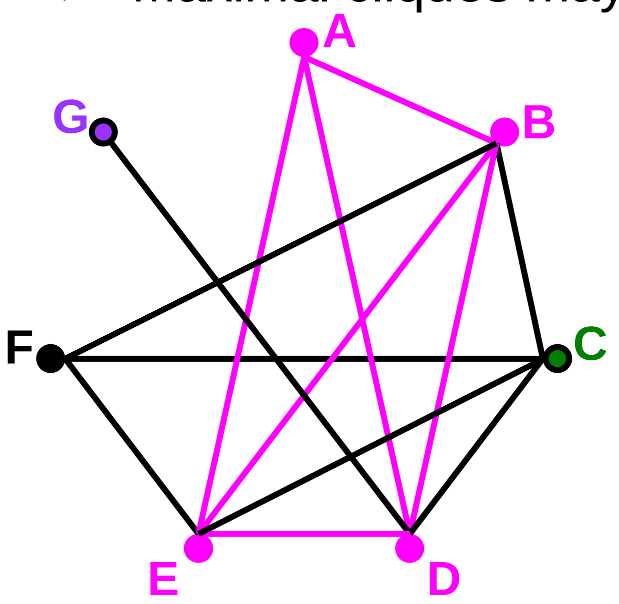


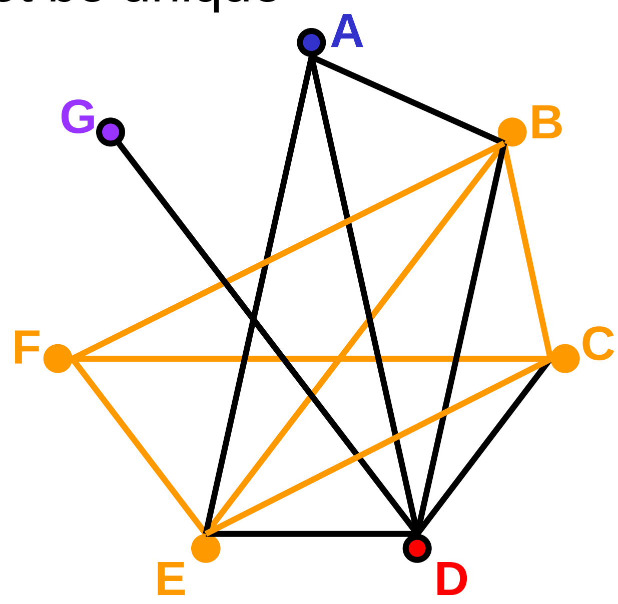


- A **maximal clique** is not contained in any larger clique

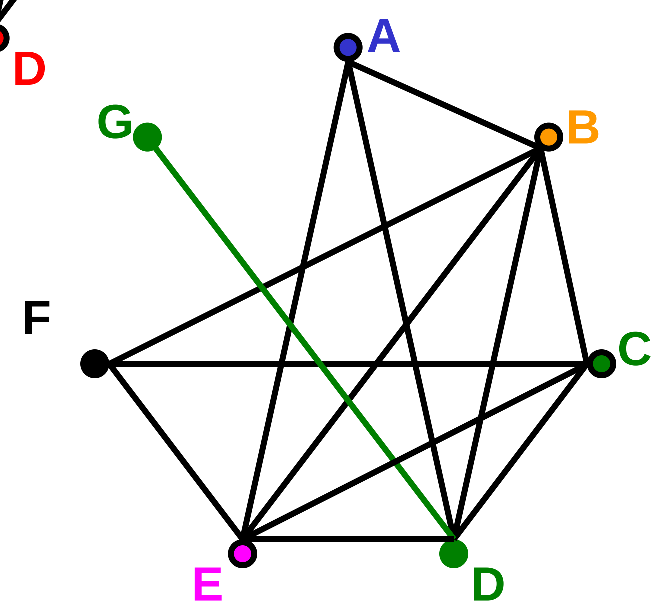
Maximal Cliques

- Maximal cliques may not be unique



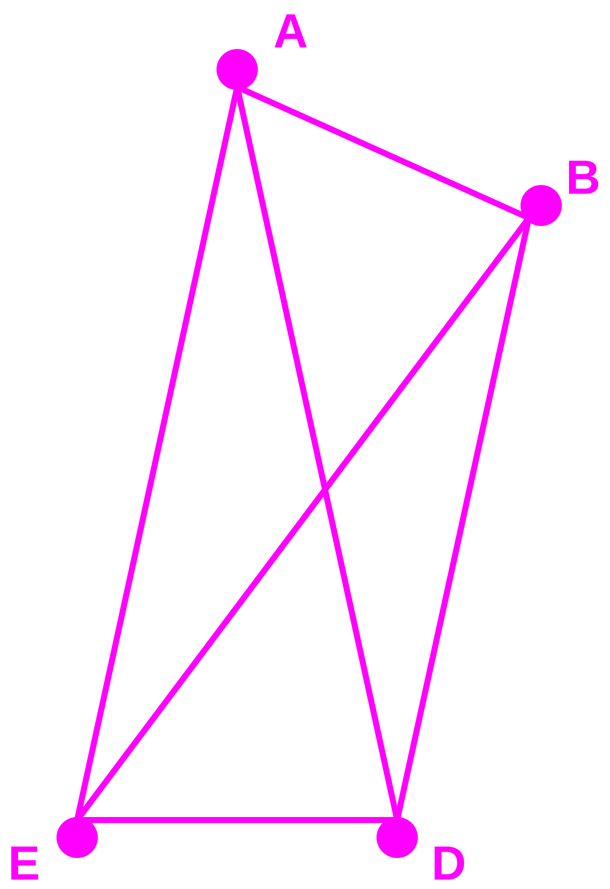


- Maximal cliques can be different sizes



Finding Maximal Cliques

- Finding all maximal cliques is a very difficult problem (NP-hard)
- Bron and Kerbosch created first efficient search technique in 1973
- Koch and others have improved efficiency
- Algorithms often based on technique called backtracking
- All maximal cliques can typically be found for problems that appear in practice



Meals Ready to Eat

Example from collaborative research with Michael Nestrud

Meals Ready to Eat (MREs)

MREs and Combinatorial Tools

- MREs comprised of several components
- Number of component combinations vast
- Resources for choosing combinations limited
- Combinatorial tools offer many potential benefits
  - Can discover optimally acceptable MREs
  - Can determine optimal portfolios of MREs

Determining Compatibility

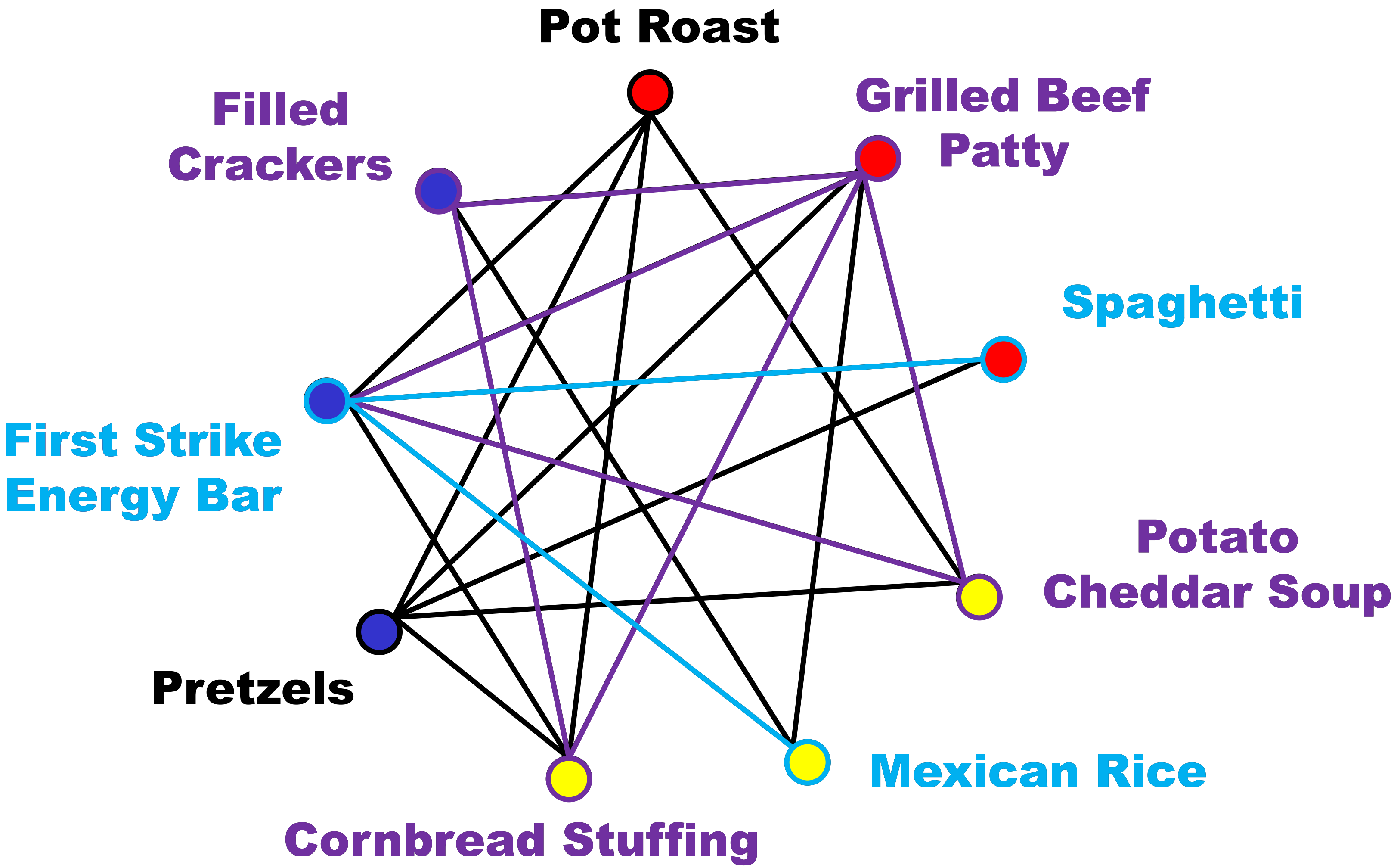
Entrée
Grilled Beef Patty
Pot Roast
Spaghetti with Meat Sauce

Starch
Potato Cheddar Soup
Mexican Rice
Cornbread Stuffing

Side
Pretzels
First Strike Energy Bar
Filled Crackers

Appropriate to combine in an MRE Menu?		Y	N
Pot Roast	Stuffing	X	
Pot Roast	Potato Soup	X	
Pot Roast	Mex Rice		X
Pot Roast	Pretzels	X	
Pot Roast	First Strike	X	
Pot Roast	Filled Crackers		X
…	…		

Finding Maximal Cliques



Maximal Cliques = Optimized MRE Menus

	Entrée	Starch	Side
1	Roast	Potato	Pretzels
2	Roast	Potato	First Strike
3	Roast	Stuffing	Pretzels
4	Roast	Stuffing	First Strike
5	Beef	Potato	Pretzels
6	Beef	Potato	First Strike
7	Beef	Potato	*Cracker*

	Entrée	Starch	Side
8	Beef	Rice	First Strike
9	Beef	Rice	*Cracker*
10	Beef	Stuffing	Pretzels
11	Beef	Stuffing	First Strike
12	Beef	Stuffing	*Cracker*
13	Spaghetti	Potato	Pretzels
14	Spaghetti	Potato	First Strike

Example – Pizza Optimization

Example from collaborative research with Michael Nestrud

Discovering Optimal Pizzas

- 200 respondents each given 300 yes/no questions:

Would you consume the following items together on a pizza?		Y	N
Mushroom	Ham	X	
Ham	Ground Sausage	X	
Italian Sausage	Jalapeno		X
Jalapeno	Italian Sausage	X	
Ground Sausage	Mushroom	X	
Broccoli	Pineapple		X
…	…		

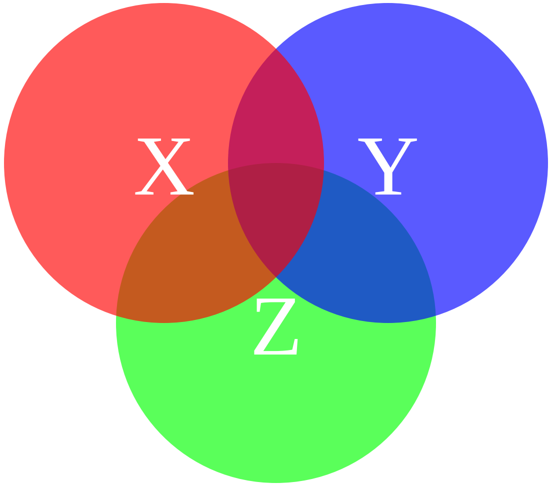
Discovering Optimal Pizzas

- We want maximal cliques of size 5 but none of size 6
- We find a threshold that gives cliques of size 4 but none of size 5
- Any larger threshold creates cliques of size 6
- We have 25 maximal cliques = 25 optimal pizzas
  - 8 with 4 toppings
  - 3 with 3 toppings
  - 2 with 2 toppings
  - 12 with 1 topping
- From these 25 pizzas we want 5 that together reach as many consumers as possible
- To find best combination we use maximal coverage techniques

Maximal Coverage

Total Unduplicated Reach and Frequency

- TURF (Total Unduplicated Reach and Frequency) is a technique introduced by Miaoulis, Parsons and Free
  - Venn Diagram based approach



  - Maximizes total coverage of combinations
  - Originally used to estimate reach of advertising
  - Has been used extensively to maximize purchase interest

Total Unduplicated Reach and Frequency

- Goals:
  - Find combination that maximizes total reach

    and/or

  - Find combination that maximizes total frequency

- Idea:
  - Assign consumers to products or concepts
  - Find combination covering most consumers
  - Consider duplication to avoid double counting

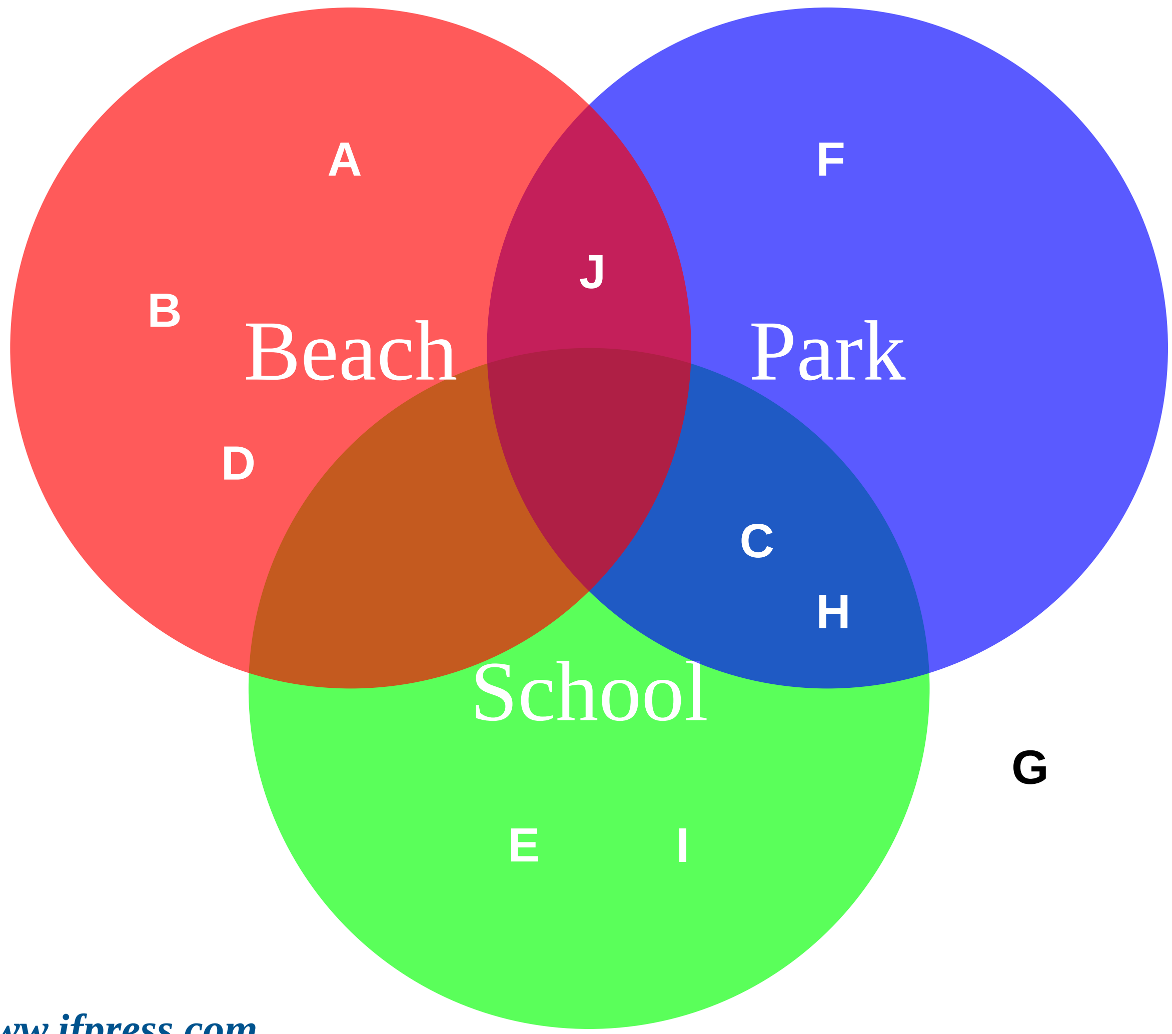
TURF – Example

- Three advertising concepts for juice drink product
- Ten consumers polled

Consumer	Beach	Park	School
A	x		
B	x		
C		x	x
D	x		
E			x
F		x	
G			
H		x	x
I			x
J	x	x	

TURF – Example

- Goal: Find two concepts that reach most consumers



Beach + **Park** = 7

Beach + **School** = 8

Park + **School** = 6

TURF – Advantages and Disadvantages

- **Advantages**
  - Flexible technique with many applications
  - Easy to understand and explain
  - Clear guidance
  - Visualization possible for small number of concepts
- **Disadvantages**
  - Visualization not possible for larger numbers of concepts
  - TURF provides no mathematical contribution
  - Huge number of combinations when number of concepts is large
    - 5 concepts out of 100 ≈ 75,000,000 combinations
    - 10 concepts out of 100 ≈ 17,300,000,000,000 combinations
- Modern maximal coverage techniques minimize disadvantages

Example – Pizza Menu Optimization

Example from collaborative research with Michael Nestrud

Pizza Menu Optimization

- Goal: Create 5 pizzas with up to 5 toppings each
- We found 25 optimal pizzas using cliques
- Want a menu of 5 optimal pizzas with maximal customer reach
- 53130 possible menus
- Options:
  - Can maximize the number of ingredients appearing on menu
  - Can maximize the number of consumers predicted to like at least one of the pizzas on the menu
  - Can poll consumers directly regarding the 25 optimal pizzas and then find 5 pizzas with maximal reach

Follow Up Study

- Each consumer is asked whether or not they would consume each of the 25 optimal pizzas

Would you consume a pizza with the following toppings?				Y	N
Ricotta Cheese	Tomato	Pepperoni	Italian Sausage	X	
Tomato	Chicken	Roasted Garlic	Mushroom	X	
Chicken	Tomato	Basil	Roasted Garlic		X
Basil	Tomato	Roasted Garlic	Italian Sausage	X	
Ground Sausage	Roasted Garlic	Tomato	Italian Sausage	X	
Italian Sausage	Pepperoni	Roasted Garlic	Tomato		X
…	…	…	…		

Results

Consumer	Pizza 1	Pizza 2	Pizza 3	…
1		x	x	…
2	x			…
3		x		…
4	x			…
5	x		x	…
6		x		…
7	x			…
8		x	x	…
9				…
10	x	x	x	…
…	…	…	…	…

Finding an Optimal Menu

- Using maximal coverage we find 5 pizzas that cover 92% of consumers:
  - **Tomato**, **Roasted Garlic**, Chicken and Basil
  - Onion, Ricotta Cheese, **Italian Sausage** and Pepperoni
  - **Italian Sausage**, Ground Sausage, **Roasted Garlic** and **Tomato**
  - Eggplant, Broccoli and Artichoke
  - Bacon, Ham and Red Onion

- Number of possible menus: 12,500,000,000,000,000,000,000

- Optimal menu was obtained using 300 initial questions and 25 follow-up questions

- All questions were “yes/no”

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