Optimal Rotations for Incomplete Designs

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Background: Designs for sequential monadic consumer product research, whether complete or incomplete, are subject to compromise due to attrition during testing. So, no matter how perfectly balanced a design may be at the outset, the final data will inevitably reflect a deviation from perfection. In previous technical reports[1,2] we have discussed different approaches to accounting for position, sequential effects and sequence spread in sequential monadic (complete block) designs and argue for rotations that minimize the impact of these variables at the outset of testing so that their effects on bias and variance in the final analyses can be minimized. Note that minimizing these effects is essential whether one expects attrition to be an issue or not. We recommended the use of Column Randomization and Search (CR&S) to create optimal rotations and showed how this approach is superior to either within subject randomization or replicated Williams Squares[3]. CR&S is a computer intensive method that considers millions of possible designs and chooses a design that minimizes the variances in the counts of products by position, by paired sequences, and by sequence spread throughout the design. In this technical report we explore rotations for incomplete designs.

Scenario: Your consumer research on personal care products routinely involves monadic evaluations of a set of samples, each evaluated by carefully matched consumer cells of approximately 200 respondents. While this approach generates valuable information on individual products, it lacks the insights that can be uncovered using a sequential monadic design and an unfolding analytical technique such as Landscape Segmentation Analysis.®[4] To that end, you decide to experiment with a sequential monadic design for a set of 7 shampoo and conditioner bundles. For this type of project, respondents are asked to use each bundle for at least 5 days over a one-week period. A complete block approach would require that each respondent participate in 7 successive weeks, or almost 2 months. To limit participant attrition and the potential for reduced data quality that can occur in research involving extensive data collection time periods, you decide to consider a balanced incomplete block design (BIBD), with respondents evaluating 4 of the 7 bundles. You would like each bundle to be evaluated about 200 times. You plan on recruiting 378 respondents, which will result in a final sample size of 350 (assuming about an 8% attrition) with a total number of evaluations of about 200 per bundle. This number will provide the ability to study potential population segmentation and identify the category's Drivers of Liking® using techniques, such as Landscape Segmentation Analysis,® which extract richer and more accurate information using multiple sample evaluations per respondent.

Incomplete Designs: Incomplete block designs were originally developed to improve precision in situations where there is significant within-block variation that could be confounded with treatment effects. The idea behind these designs is that treatment comparisons can be made with more precision when a smaller number of the treatments are contained within each block to restrict the effect of within-block variation. These designs apply, for instance, to crop field trials when there is a difference in fertility or drainage across the block. In consumer product testing the participants form the blocks and it is sometimes desirable to limit the number of products tested by each subject, especially when it can be expected that there will be large within-subject effects, manifested with more products tested per person. A common misconception is that incomplete block designs will result in lower costs due to lower per subject incentives. In fact, the overall sample size will need to be increased, which will increase recruitment costs, to maintain the same number of evaluations per product as a complete block design. This is necessary to ensure sufficient power in treatment comparisons. Consequently, cost savings, if any, will be minimal, and we generally recommend a complete block design when possible.

Like complete block designs, incomplete block designs are subject to imbalances and pose special challenges in creating rotations that account for position, sequence, and sequence spread. In agricultural experiments, sequential order does not occur as it does in consumer research, therefore published designs of the method[5] do not account for it. The flexibility of the CR&S method is particularly valuable in helping to reduce bias and variance in study designs for consumer testing.

In our previous technical report[2], we discussed the situation that arises when sequences are only important within a day, such as in food sensory testing, and sequences from one day to the next are of no interest due to a time delay. One way of creating an incomplete design is to use the CR&S method to generate a design as if testing occurred over separate time intervals. For a particular participant, the treatments in the incomplete block are placed in the first time interval and all remaining treatments are assigned by themselves to the remaining time intervals. Although column randomization occurs over all the columns and thus includes all of the possible products, the design chosen considers only the treatments that occur in the first interval. This approach removes the remaining treatment sequences from consideration. This method will provide a design for the incomplete blocks

	1st Bundle	2nd Bundle	3rd Bundle	4th Bundle
Cons 1	C	E	F	G
Cons 2	A	D	G	F
Cons 3	G	A	B	E
Cons 4	F	B	A	C
Cons 5	D	G	C	B
Cons 6	E	C	D	A
Cons 7	B	F	E	D

Figure 1. Balanced incomplete block design for 7 consumers evaluating 4 of 7 samples.

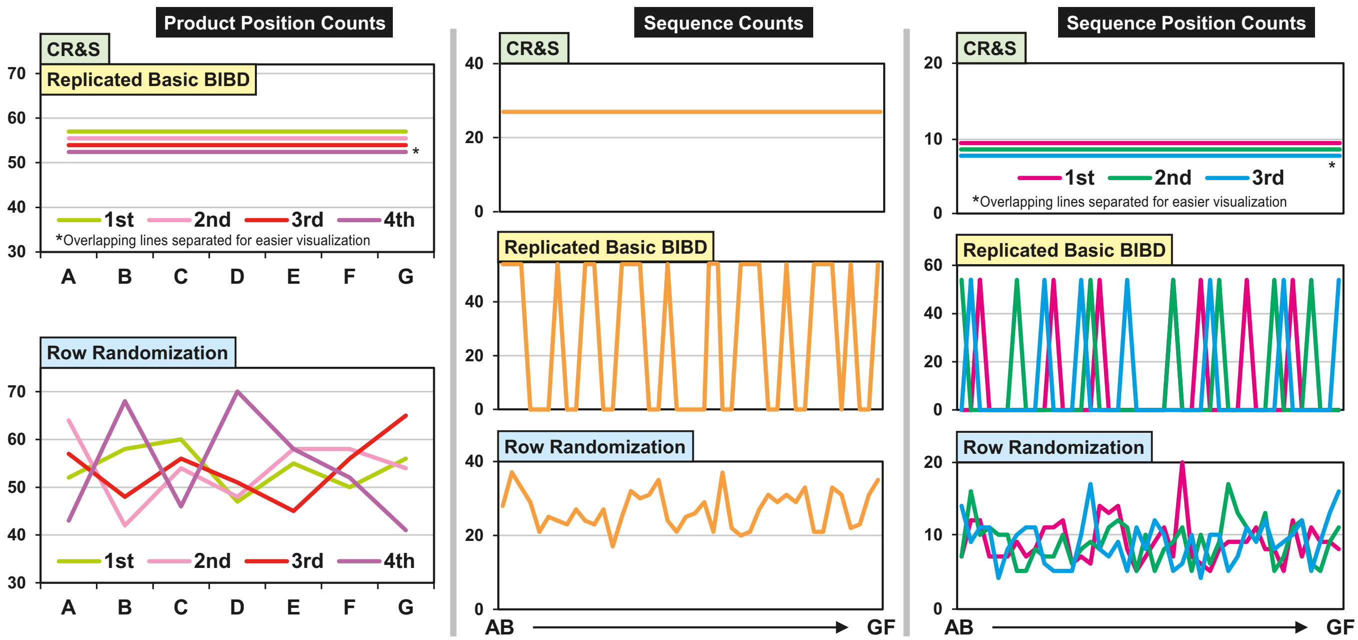

Figure 2. Product position, sequences, and sequence position counts for three incomplete design generation methods.

optimized over position, sequence and sequence spread within the incomplete portion of the total product set.

Incomplete Block Design Considerations: You must decide how to design your sample presentation rotations to minimize bias and variance associated with positions, sequences, and sequence positions across the incomplete design. You are aware of the CR&S method which minimizes those biases when considering a complete block design. However, you will need to adapt this method when the design is incomplete. Cochran & Cox[5] provide a series of BIBDs on various subsets of a given number of treatments. You find the basic design for 4 out of 7 treatments. Each treatment appears four times overall and appears with each other treatment the same number of times (2). This design does not account for position effects without rearrangement. Figure 1 shows a rearrangement to account for position (but not sequences) across the basic design. A balanced position arrangement may not be feasible for other designs. You could use this basic design and repeat it 54 times to obtain your full set of rotations (before attrition). However, this would not ensure the balance of sequences and sequence spread, although it would balance for product positions.

Creating the Bundle Design: For comparison, you first replicate the design in Figure 1 a total of 54 times leading to 378 rows. Figure 2 shows the frequencies with which the sequences occur. This basic design does very poorly in terms of addressing sequences and sequence spread which is important because you will not have a buffer evaluation period with a neutral sample to reduce the carryover effect of one bundle to another. This is analogous to replicating a Williams Square in a complete block design.[3] Another approach you consider is to randomize each row of the 378 × 4 matrix, with the hope that it will generate a more balanced solution. Although you lose position balance the sequence counts are more even, but this design is clearly not optimal either as seen in Figure 2. To generate a balanced design using the CR&S method, some experimentation reveals that your final design must contain a multiple of 42 blocks. Starting with 378 participants, which is a multiple of 42, you can begin your data collection with perfect position, sequence, and sequence balance as shown in Figure 2. With a rate of 8% random dropouts, you will end with a sample close to 350 and close to ideal balance.

Conclusion: Incomplete designs for consumer testing are subject to the effects of position and sequences in a manner similar to complete designs. However, they are also more complicated to ensure that these effects do not compromise results due to bias and increased variance. The CR&S method may be valuable to design incomplete block experiments by treating their design as part of a complete block experiment in which positions within only a portion of the design are of interest to control for position, sequence and sequence spread effects. Software to choose these designs is available in ***IFPrograms***®.[6]

References

1. Ennis, D. M., Rousseau, B, and Ennis, J. M. (2014). Rotations in product tests and surveys. *IFPress*, **17**(1) 3-4.

2. Rousseau, B. and Ennis, D. M. (2021). Generating optimal sample presentation orders. *IFPress*, **24**(1) 3-4.

3. Williams, E. J. (1949). Experimental designs balanced for the estimation of residual effects of treatments. *Australian Journal of Scientific Research,* Ser. A 2, 149-168.

4. Ennis, D. M. and Rousseau, B. (Eds.) (2020). *Tools and Applications of Sensory and Consumer Science*, Parts 4 & 5, pp. 72-117. Richmond, VA: The Institute for Perception.

5. Cochran & Cox (1992). *Experimental Designs.* New York: Wiley.

6. *IFPrograms*® software, Tools version, The Institute for Perception, Richmond, VA.