ASTM Meeting
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Vancouver, British Columbia

Multiplicative Comparisons of Normal Variables

Presented By:
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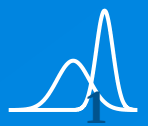
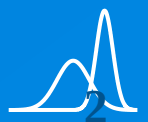


Ratio and Multiplicative Comparisons

Publications

00. Ennis, Ennis (*Submitted)*. Confidence Bounds for Multiplicative Comparisons. CIS

85. Ennis *et al.* (2008). Confidence Bounds for Positive Ratios of Normal Random Variables. CIS, 37, 307-317



Typical Ratio Statements

Compared to a competitor…

- Carpet treatment **reduces** malodor **five times better**
- Tooth whitening treatment **is twice as effective**
- Air freshener **lasts 20% longer**
- Cleaning product **performs "up to 30%" better**

- *Are these actually ratio statements?*

Ratio vs. Multiplicative Statements

Ratio statements

- Interpreted as **X**/**Y** > **c**
- Problems if **X** or **Y** is negative
- Consider P(**X**/**Y** > **c** | **Y** > 0)
- Details in Ennis *et al.* (2008)
- Extension of Fieller (1932)

Multiplicative statements

- Interpreted as **X** > **cY**
- Only meaningful when **X** is positive
- Consider P(**X** > **cY** and **X** > 0)
- Details in Ennis and Ennis (*Subm.*)
- Extension of Ennis *et al.* (2008)

Examples of Multiplicative Statements

Compared to a competitor…

- Carpet treatment **reduces** malodor **five times better**
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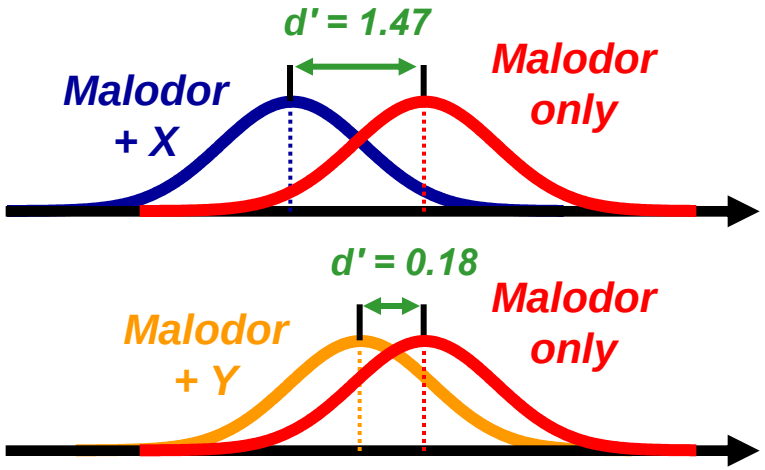
Example: Malodor Reduction

- 100 consumers perform two 2-AFCs between two odor test chambers:

	Odor test chamber A	Odor test chamber B
Comparison 1	Malodor only	Malodor + Air Freshener X
Comparison 2	Malodor only	Malodor + Air Freshener Y

- **Question**: Which of the two chambers has more malodor?
- Order of presentation and evaluation balanced over the whole design
- Use *d'* values to work with differences on an interval scale

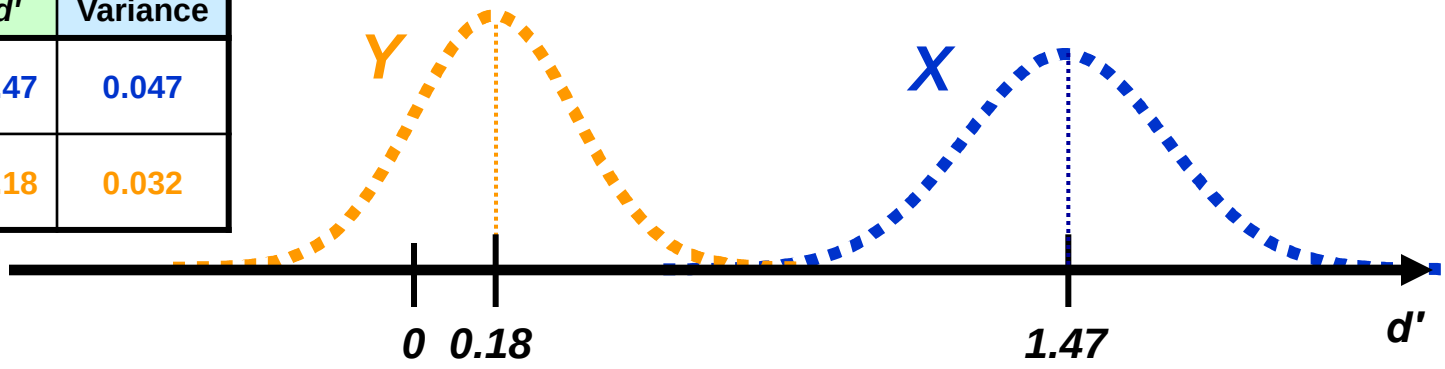
Condition	# times selected Malodor only	N	d'	Variance
Malodor + Air Freshener X	85	100	1.47	0.047
Malodor + Air Freshener Y	55	100	0.18	0.032

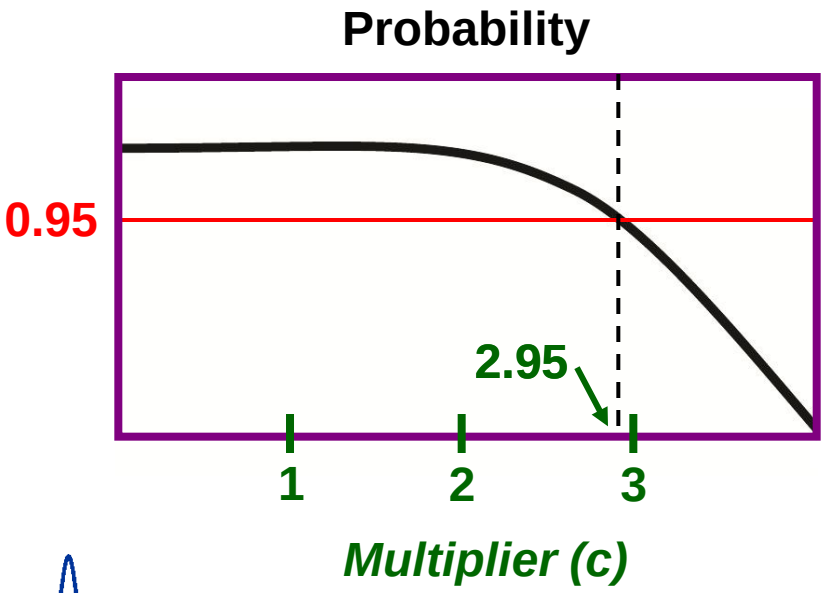


- Point estimate of multiplier: **1.47**/**0.18** = 8.17

Example: Malodor Reduction

Condition	d'	Variance
Malodor + Air Freshener X	1.47	0.047
Malodor + Air Freshener Y	0.18	0.032





- For each value of **c**
  - Compute P(**X** > **c**Y and **X** > 0)
  - Determine if P > 0.95
- Largest **c** value
  - Multiplicative: **2.95**
  - Ratio: **2.85**

Comparison of Methods

- Six cases involving 2-AFC method
- Proportions correct for treatments vs. control are **0.64** and **0.52**
- 95% confidence bounds:

Sample Size	Ratio	Multiplicative
100	0.91	1.08
125	1.10	1.26
150	1.24	1.39
200	1.47	1.62
300	1.86	1.99
500	2.33	2.44

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