Joint Statistical Meetings

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Lower Bounds for Multiplicative Comparisons

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Comparative statements

Compared to a competitor…

Carpet treatment reduces malodor **five times better**

Tooth whitening treatment is **twice as effective**

Air freshener lasts **20% longer**

Cleaning product performs **up to 30% better**

What is statistical justification?

Ratio approach

Ennis et al. (2008). Confidence Bounds for Positive Ratios of Normal Random Variables. CIS, 37, 307-317

$X/Y > c > 0$

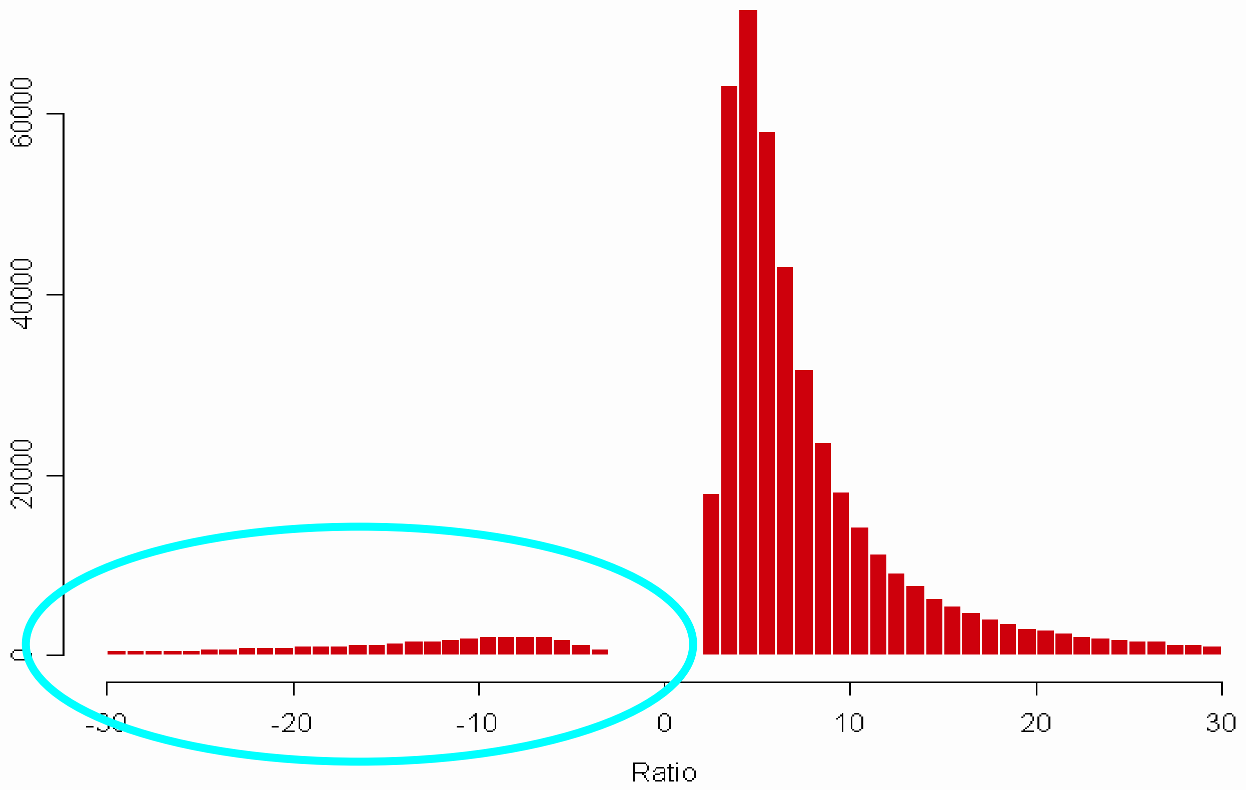
Extends Fieller (1932)

Conditioned on *Y* positive

Consider $P(X/Y > c \mid Y > 0)$

Problem with ratio approach

Competitive advantage lost when $X > 0$ and $Y < 0$



Multiplicative approach

Ennis, J. and Ennis D. Confidence Bounds for Multiplicative Comparisons. CIS (*Submitted)*

$X > cY$, $c > 0$

Improves Ennis et al. (2008)

No conditions on Y

Consider P(X > cY and X > 0)

Finding a lower confidence bound

To find a lower (1- α) confidence bound we solve

$$P(X > cY \text{ and } X > 0) = 1 - \alpha$$

Note that P($X > cY$ and $X > 0$) can be computed as

$$\int_0^\infty \int_0^\infty f(\mathbf{x})\,d\mathbf{x}$$

using P($X > cY$ and $X > 0$) = P($X - cY > 0$ and $X > 0$)

A single integral expression

P($X > cY$ and $X > 0$) can also be computed using a single integral expression (c.f. Ennis et al. (2008))

$$\int_0^\infty \int_0^\infty f(\mathbf{x})d\mathbf{x} = \int_0^\rho g(\mu_1, \mu_2; t)\,dt + \Phi(\mu_1)\Phi(\mu_2),$$

where $$\rho = \frac{\sigma_x^2 - cCov_{xy}}{\sqrt{(\sigma_x^2 + c^2\sigma_y^2 - 2cCov_{xy})\sigma_x^2}},$$

$$\mu_1 = \frac{\mu_x - c\mu_y}{\sqrt{\sigma_x^2 + c^2\sigma_y^2 - 2cCov_{xy}}}$$ and $$\mu_2 = \frac{\mu_x}{\sigma_x}$$

Malodor reduction example

Two groups of 100 consumers

Each consumer performs a single 2-alternative forced choice (2-AFC) trial

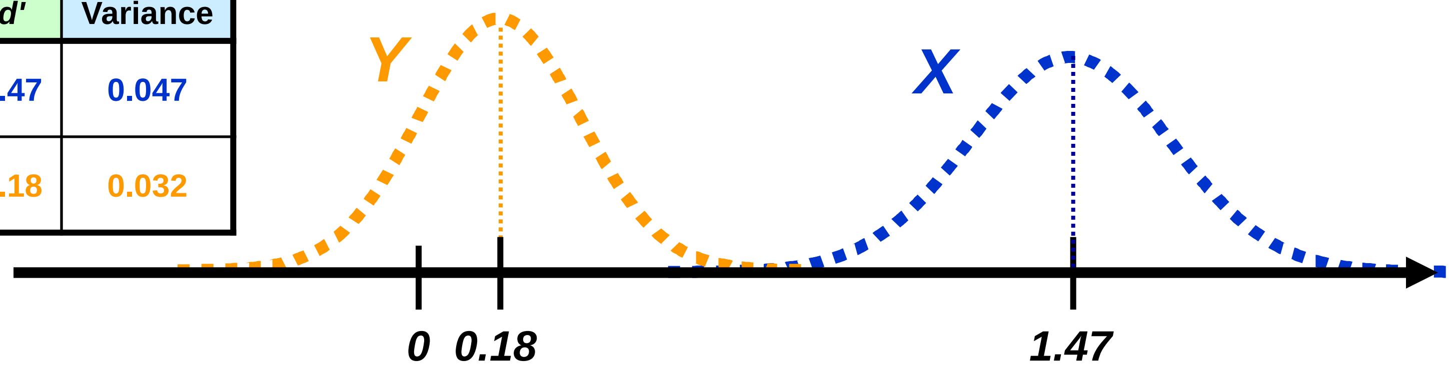
Condition	Frequencies	*d'*	Variance
Malodor / Malodor + *X*	**85 / 15**	**1.47**	**0.047**
Malodor / Malodor + *Y*	**55 / 45**	**0.18**	**0.032**

Convert to d′ to obtain differences on an interval scale (Thurstone 1927)

Variance in estimates can also be calculated using maximum likelihood

Malodor reduction example

Condition	*d'*	Variance
Malodor + Air Freshener *X*	1.47	0.047
Malodor + Air Freshener *Y*	0.18	0.032

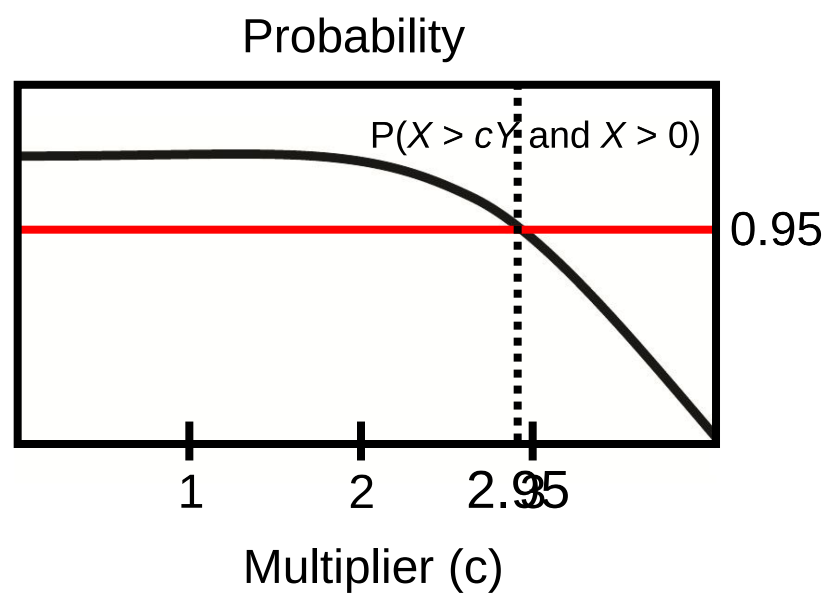


Consider P($X > cY$ and $X > 0$)

Multiplicative:

Ratio: 2.85

Point Estimate: 8.17 = 1.47/0.18



Ratio vs. multiplicative statements

Ratio statements

Interpreted as $X/Y > c > 0$

Conditioned on Y positive

Consider $P(X/Y > c \mid Y > 0)$

Details in Ennis *et al.* (2008)

Extends Fieller (1932)

Multiplicative statements

Interpreted as $X > cY$, $c > 0$

No conditions on Y

Consider $P(X > cY$ and $X > 0)$

Details in Ennis and Ennis (*Subm.*)

Improves Ennis *et al.* (2008)

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Thank You

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