Optimal Rotations to Balance Sequential Effects

Will Russ, Kevin Yang and Daniel Ennis

Background: In previous technical reports[1] we discussed methods for developing improved rotations for sequential testing to remove bias and lower testing variances. The benefit of doing this is to obtain more accurate information and to create more powerful tests with the opportunity to lower testing cost. For most designs, the use of the Column Randomization and Search (CRAS)[1] method has proven to be useful for many designs including complete block, incomplete block, and testing with sequences containing interruptions, such as testing over several days. This method searches for position, sequences and the location of sequences in the design (called sequence spread) that are not often accounted for in the experimental design of product tests, other types of surveys and clinical trials. When they are considered, it is common to use Williams Squares[2] to control for position and sequences but not sequence spread. For instance, if the sequence (1 2) occurs at the beginning and end of a Williams Square with multiple products, this sequence will never occur at the middle of the test and replication will perpetuate this condition. This could be important because performance from the beginning to the end of a series of evaluations changes and could be different for the sequences as well as the products themselves. These designs are replicated when the number of rotations needed exceeds the size of the design. It is possible to overcome this limitation of replicated Williams Squares so that designs based on them can handle sequence spread. The purpose of this technical report is to explore this possibility.

A Replicated Williams Square with Five Products: Table 1 shows an example of a Williams Design with 5 products labeled 1 to 5. There are ${}^5C_2$ combinations of pairs of products and twice that number, or 20, if forward and reverse orders are considered. Table 1 shows that there are 10 rows because of the need to meet the requirement that the 5 positions and 20 sequences are accounted for at least once. In this case they each appear twice. But notice in Table 2 that the sequences, e.g. (1 2) are not spread equally across the design.

Scenario: You are a data scientist working in a company that markets air-care products to homeowners. Sequential effects in the evaluation of fragrances often arise and you would like to minimize bias and improve precision in central location tests. Some of your suppliers simply randomize the presentation order and you are aware that this practice does not account for position or sequence effects properly. Lately you have considered using Williams Square designs. These designs are set up so that each product appears an equal number of times in each position. They also ensure that there is balance in the frequency with which the sequences occur but not their occurrence in the rotations. To improve this condition it would be necessary, when replicating the design, to consider making sequence selections with the intent of ensuring a balance in the sequence spread.

1	2	5	3	4
2	3	1	4	5
3	4	2	5	1
4	5	3	1	2
5	1	4	2	3
4	3	5	2	1
5	4	1	3	2
1	5	2	4	3
2	1	3	5	4
3	2	4	1	5

Table 1. A Williams square with 5 products and 10 rotations. Each pair and its reverse appears twice in the design and each product appears an equal number of times in each position. However, the occurrence of sequences in the design is not uniform as shown for the sequence (1 2).

Balancing Sequence Spread: Table 2 shows the frequency of occurrence of sequences across the design of Table 1 and Figure 1 is a visual guide to Table 2. Notice that the frequency of occurrence of sequences in particular locations form a definite pattern. A design with 10 rotations appended to one that is the complement of the pattern in Figure 1 would solve the sequence spread limitation for this design.

	Item Sequence Spread			
Item Sequence	1-2	2-3	3-4	4-5
(1 2)	1	0	0	1
(1 3)	0	1	1	0
(1 4)	0	1	1	0
(1 5)	1	0	0	1
(2 1)	1	0	0	1
(2 3)	1	0	0	1
(2 4)	0	1	1	0
(2 5)	0	1	1	0
(3 1)	0	1	1	0
(3 2)	1	0	0	1
(3 4)	1	0	0	1
(3 5)	0	1	1	0
(4 1)	0	1	1	0
(4 2)	0	1	1	0
(4 3)	1	0	0	1
(4 5)	1	0	0	1
(5 1)	1	0	0	1
(5 2)	0	1	1	0
(5 3)	0	1	1	0
(5 4)	1	0	0	1

Table 2. Occurrence of sequences in the design of Table 1.

Selecting the Optimal Design: By considering the structure of Table 2, you can determine a lower bound on the number of subjects needed to achieve perfect balance. A design which fills each cell with a 1 will have a variance of zero for position, sequence, and sequence spread. In the case of 5 products, you would need 20 respondents, twice the amount in the Williams

	Item Sequence Spread			
Sequence	1-2	2-3	3-4	4-5
(1 2)	■			■
(1 3)		■	■	
(1 4)		■	■	
(1 5)	■			■
(2 1)	■			■
(2 3)	■			■
(2 4)		■	■	
(2 5)		■	■	
(3 1)		■	■	
(3 2)	■			■
(3 4)	■			■
(3 5)		■	■	
(4 1)		■	■	
(4 2)		■	■	
(4 3)	■			■
(4 5)	■			■
(5 1)	■			■
(5 2)		■	■	
(5 3)		■	■	
(5 4)	■			■

A

	Item Sequence Spread			
Sequence	1-2	2-3	3-4	4-5
(1 2)		■	■	
(1 3)	■			■
(1 4)	■			■
(1 5)		■	■	
(2 1)		■	■	
(2 3)		■	■	
(2 4)	■			■
(2 5)	■			■
(3 1)	■			■
(3 2)		■	■	
(3 4)		■	■	
(3 5)	■			■
(4 1)	■			■
(4 2)	■			■
(4 3)		■	■	
(4 5)		■	■	
(5 1)		■	■	
(5 2)	■			■
(5 3)	■			■
(5 4)		■	■	

B

Figure 1. Sequence spread counts for the design in Table 1 (A) and its complement (B).

Design. More generally, as each respondent contributes one cell to each column, you would need $n * (n - 1)$ respondents where n is the number of products to be evaluated. Finding an actual optimal design is a challenging task due to the combinatorial nature of the problem, however, solutions have been found for many practical problem sizes. Table 3 shows one such design for 5 products.

Application to the Fragrance Study Design: You apply this approach to the Williams Design in Table 1 and obtain Table 3. This table can now be replicated and used in the fragrance CLT. The plan is to conduct this research with 300 recruited consumers, over-recruited to ensure at least 300. There are 15 sets of 20 to make up 300 rotations. However, there may be no-shows resulting in a sample size different from 300 and not an integer multiple of 20. In order to minimize the impact of a partial plan, you now consider how the partial sample should be drawn.

Rationale for Selecting the Final Design Fragment: As the final exact number of respondents is unknown, it is desirable to have an ordered design where each subsequent row keeps the cumulative variance as low as possible. Considering the counts, such as those in Table 2, along with graph theoretic tools, it is possible to find such an order that minimizes the variance of sequences or the variance of sequence spread, but not necessarily both. Since the effect of unbalanced sequences is expected to be larger than that of sequence spread, you choose to find a design which minimizes the former. These designs can be based on Williams Designs and in this case the design has 10 rows. You append these 10 rows to your original 300 and now have flexibility in the final number of respondents while maintaining low variance.

Rotation	Position 1	Position 2	Position 3	Position 4	Position 5
1	1	2	5	3	4
2	2	3	1	4	5
3	3	4	2	5	1
4	4	5	3	1	2
5	5	1	4	2	3
6	4	3	5	2	1
7	5	4	1	3	2
8	1	5	2	4	3
9	2	1	3	5	4
10	3	2	4	1	5
11	1	3	4	5	2
12	3	5	1	2	4
13	5	2	3	4	1
14	2	4	5	1	3
15	4	1	2	3	5
16	2	5	4	3	1
17	4	2	1	5	3
18	1	4	3	2	5
19	3	1	5	4	2
20	5	3	2	1	4

Table 3. A Williams design with 5 products and 10 rotations augmented with a design that is the complement of the sequence spread counts. This design perfectly controls for sequence spread with a multiple of 20 participants.

Conclusion: Rotations in tests of multiple items is a practical and important step in minimizing bias (getting more accurate responses) and improving precision in tests involving sequential monadic testing. It is not recommended to rely on randomized rotations or even replicated Williams Squares when an alternative that can account for sequence spread is available. The alternative discussed in this report provides a design that controls for all three conditions perfectly if the number of participants is a multiple of 20 and a least variance solution if the number of participants exceeds this number.

References

1. Ennis, D.M. and Rousseau, B. (Eds.) (2022). *Tools and Applications of Sensory and Consumer Science*, Parts 7. pp. 158-159; 164-167. Richmond, VA: The Institute for Perception.

2. Williams, E. J. (1949). Experimental designs balanced for the estimation of residual effects of treatments. *Australian Journal of Scientific Research*, Ser. A 2, 149-168.