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Multivariate Difference Testing with Multiple Samples

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Background: Variation in the manufacture of consumer products presents a challenge when investigating the suitability of process and formulation changes to a particular product. This issue is particularly relevant when investigating small sensory differences, like those involved in discrimination testing. In some cases, a product is best represented by several variants. The traditional discrimination protocols such as the triangular, duo-trio and *m*-alternative forced choice methods do not allow the consideration of manufacturing variation, unless multiple paired tests are performed. This approach can be costly and time-consuming. In an earlier newsletter[1], we introduced Torgerson's method of triads. Using an appropriate model, this method allows for the simultaneous estimation of multiple product differences. In this report, we discuss a multivariate model for Torgerson's method.

Scenario: Your supplier proposes an ingredient change for your strawberry flavored yogurt. Based on the findings of a previous study on vanilla flavored yogurt[1], you decide to use Torgerson's method of triads to incorporate variations between your two product lines. The current product is labeled "A", while the reformulated product is labeled "B". Subscripts on A and B refer to different production lines. In a trial a subject is given three products. An example is $\mathbf{A_1}A_2B_1$, in which the first sample ($\mathbf{A_1}$) is a reference. The task of the subject is to select one of the two alternatives (A_2 or B_1) most similar to the reference. Each subject evaluates one of each of the 12 possible triads. The results are presented in Table 1.

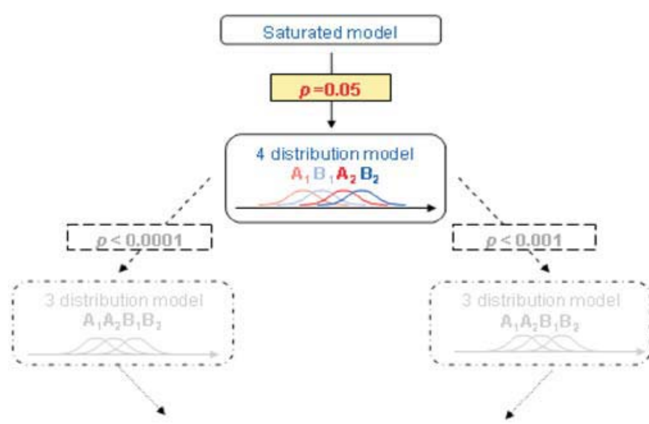
Table 1. Number of subjects choosing the first sample of the alternative pair as more similar to the reference

Triad	#	Total	Triad	#	Total
A_1 A_2B_1	25	50	B_1 A_1A_2	31	50
A_1 A_2B_2	26	50	B_1 A_1B_2	24	50
A_1 B_1B_2	33	50	B_1 A_2B_2	17	50
A_2 A_1B_1	26	50	B_2 A_1A_2	23	50
A_2 A_1B_2	26	50	B_2 A_1B_1	18	50
A_2 B_1B_2	17	50	B_2 A_2B_1	24	50

Torgerson's Method of Triads: This method was first proposed by Torgerson[2] and a unidimensional Thurstonian model has been published[3]. A multivariate model has been developed based on the fact that the method has features in common with preferential choice[4]. A Thurstonian model for Torgerson's method of triads, preferential choice and the traditional discrimination methods all involve a common set of assumptions. These assumptions are that mental entities can be represented as samples from unidimensional or multidimensional normal distributions, and that people make choices using a decision rule associated with each task. Similarities are measured in units of perceptual standard deviation and the index d´ is an estimate of a unidimensional difference parameter,

Fitting Hierarchical Thurstonian Models: In the scenario presented above, it is possible that all four products (2 products · 2 production lines) may be different. It is also possible that there are no differences due to the production lines and that the only difference is that due to the product modification. It is even conceivable that the modification has no effect. Using the method of maximum likelihood, various models are fitted to see which one best accounts for the data. Figures 1 and 2 illustrate the result of this model fitting. Figure 1 represents the unidimensional fit, while Figure 2 represents the multidimensional fit of the data. They show a hierarchical arrangement of models. At the top, the saturated model is given. This model uses up all the available degrees of freedom and provides a best fit without any interesting structure. It is useful because our goal is to develop a model with fewer degrees of freedom than the saturated model that accounts for the data as well as the saturated model.

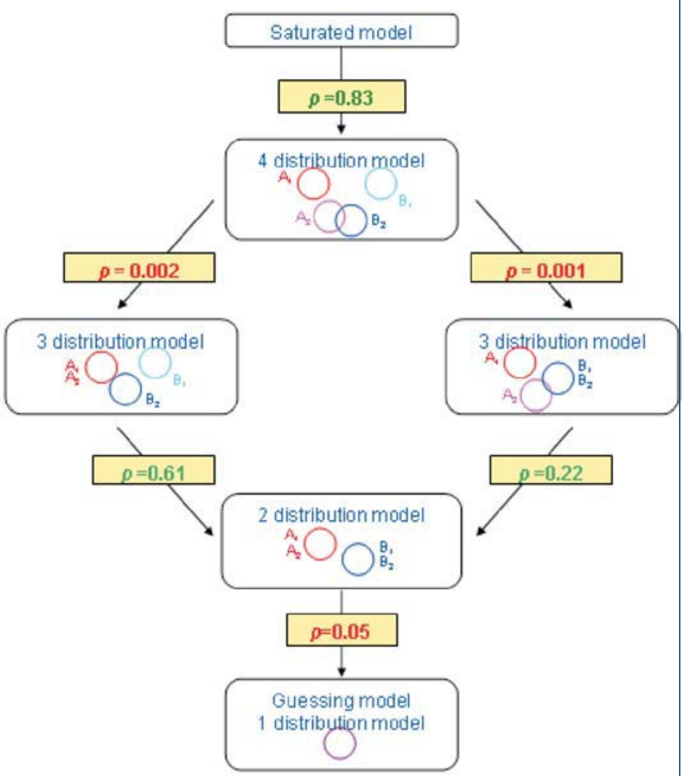
Figure 1. Hierarchical unidimensional model structure for the strawberry flavor study (the distributions are for illustration purposes only)



Calculating d´ Values with Torgerson Method: Using the data presented in Table 1, you first use the unidimensional model to fit the data. The hierarchical structure is shown in Figure 1. As can be seen, the most complex model (the one with four distributions) is significantly different from the saturated model. This means that even if we are allowed to give different means to each variant, the results are still different from the best model possible, and so the unidimensional model is not adequate. It is unnecessary to consider the lower hierarchical models since the most complex model is already significantly different from the saturated model.

You then try to fit the multidimensional model, as shown in Figure 2. The most complex model with four distributions is not significantly different from the saturated model (p=0.83), while both of the models with three distributions are significantly different from that with four distributions (p=0.002 and 0.001, respectively.) This means that the multivariate model with four distributions successfully accounts for the data, but that simpler models do not. Table 2 gives the d´ values and their variances, while Figure 3 gives a two-dimensional representation of the results.

Figure 2. Hierarchical multidimensional model structure for the strawberry flavor study (the distributions are for illustration purposes only)

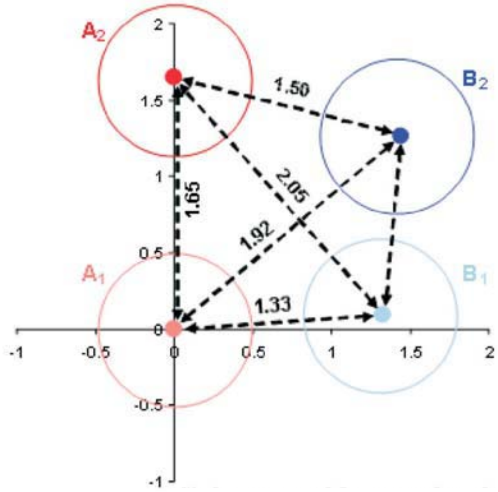


Taking into account the variation due to the production lines, the weighted average between A and B is 1.66 and is significantly different from 0. The amount of variation added by the production line is found to be 1.41, similar to that observed with the vanilla flavored yogurt (1.47)[1]. This time, the degree of change induced by the ingredient is larger than that of the previous study. Also, variation due to the ingredient change occurs on a different dimension than that induced by the production line variation. Further consumer testing is necessary in order to decide whether the sensory effect due to the ingredient change is important to consumers.

Table 2. Strawberry flavored yogurts: d´ values and *variances* for all possible product pairs (distance between the product in first column and the product in first row)

Product	A_2	B_1	B_2
A_1	1.65 *0.076*	1.33 *0.078*	1.92 *0.11*
A_2	-	2.05 *0.10*	1.50 *0.074*
B_1	-	-	1.18 *0.075*

Figure 3. Strawberry flavored yogurts: Representation of product similarities using the multidimensional Thurstonian model



Circles represent 0.5 perceptual standard deviation

Conclusion: Torgerson's method of triads has many applications in product testing. It permits the simultaneous comparison of more than two products and can be used in studies that investigate product differences. Unidimensional and multidimensional models are available, which provide measures of the similarities among products tested.

References:

1 Rousseau, B. and Ennis, D.M. (2002). Discrimination testing with multiple samples. *IFPress*, **5(1)**, 2-3.

2 Torgerson, W. S. (1958). *Theory and Methods of Scaling*. New York: Wiley.

3 Ennis, D. M., Mullen, K., & Frijters, J. E. (1988). Variants of the method of triads: Unidimensional Thurstonian models. *British Journal of Mathematical and Statistical Psychology*, **41**, 25-36.

4 Ennis, D.M. (1993). A single multidimensional model for discrimination, identification and preferential choice. *Acta Psychologica*, **84**, 17-27.