# A Thurstonian model for the Degree of Difference test with extensions to unequal variance, sequence effects and replicated data

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DTU Informatics
Department of Informatics and Mathematical Modelling

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Same-Different test:

| Same | Different |
| --- | --- |
| ☐ | ☐ |

Degree-of-Difference test:

| Same | 2 | 3 | 4 | Different |
| --- | --- | --- | --- | --- |
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# Giving answers — $\tau$ criteria and the decision rule

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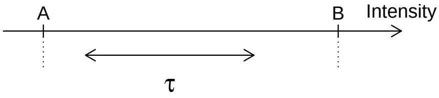
Same-Different:

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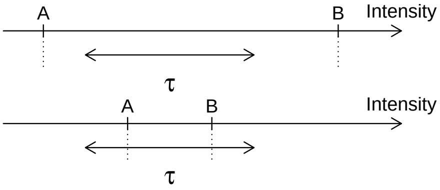


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$$|B - A| < \tau \quad \rightarrow \quad \text{"same"}$$

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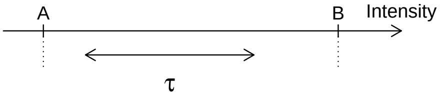


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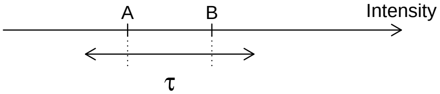
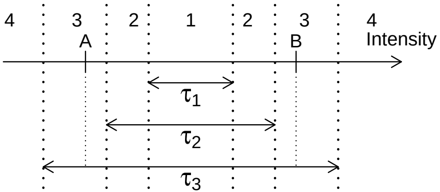



$$|B - A| < \tau \quad \rightarrow \quad \text{"same"}$$

Degree of difference:




Rating scale: 1 ☐ 2 ☐ 3 ☐ 4 ☐

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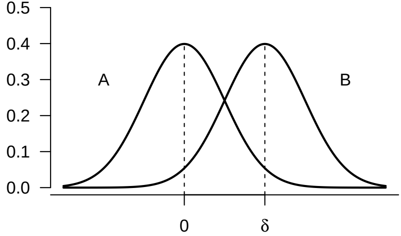


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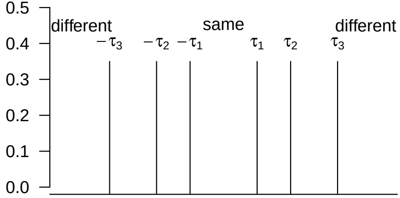

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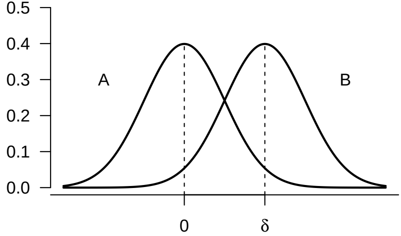


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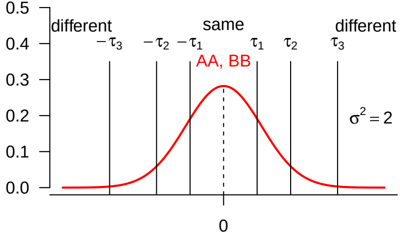

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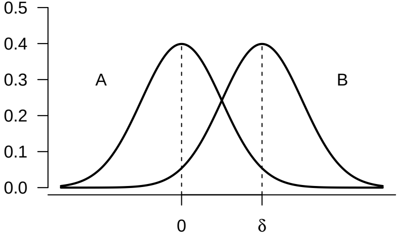


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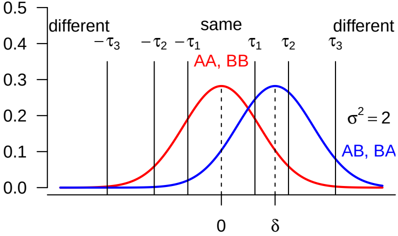

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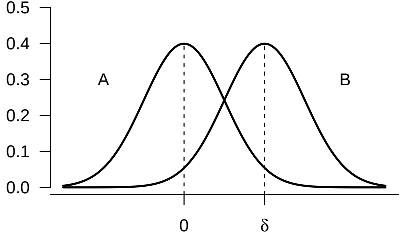


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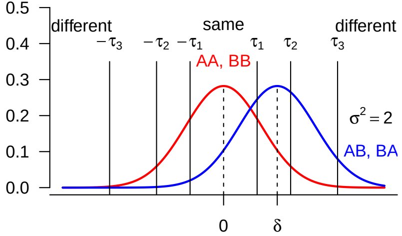


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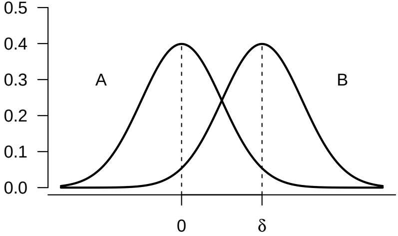


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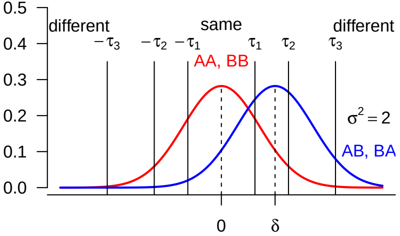


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Maximum likelihood estimation of parameters:

$$\text{likelihood} \sim f_s(\boldsymbol{\tau}) + f_d(\boldsymbol{\tau}, \delta)$$

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Response scale:

| Same | | | | | | Different |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ |

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Data obtained:

| Pair | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
|---|---|---|---|---|---|---|---|---|
| Same | 26 | 22 | 20 | 13 | 9 | 8 | 2 | 100 |
| Diff | 17 | 16 | 16 | 15 | 14 | 14 | 8 | 100 |

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- Comparison of protocols

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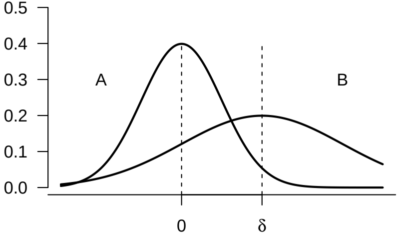

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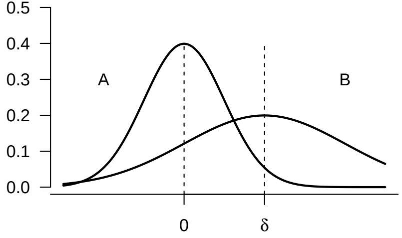


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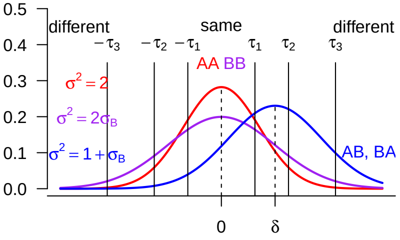

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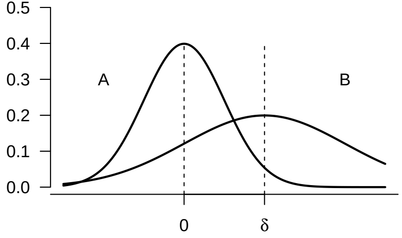


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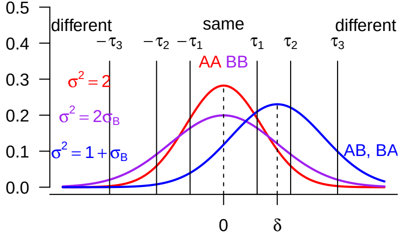


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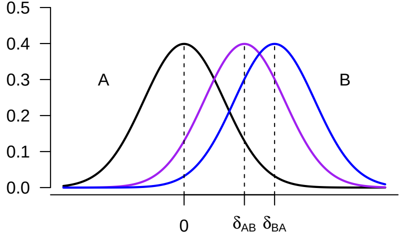

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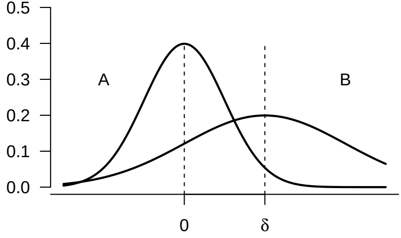


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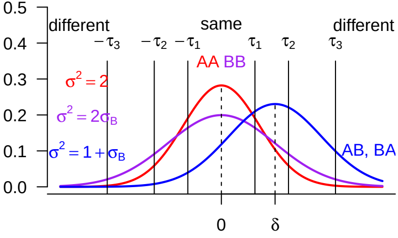


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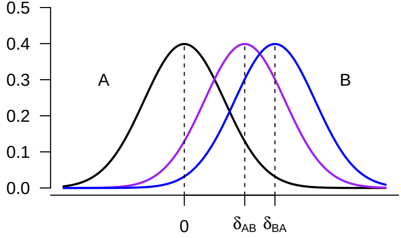


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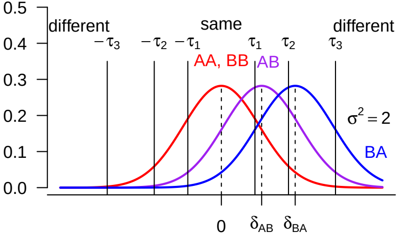

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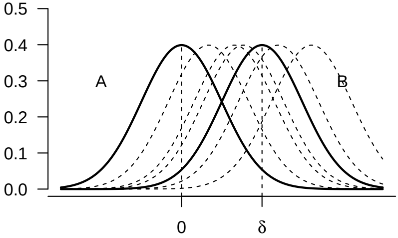


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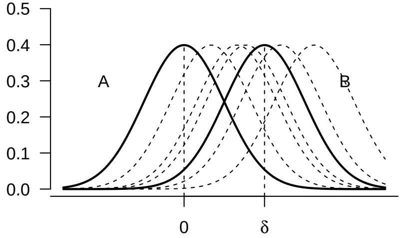


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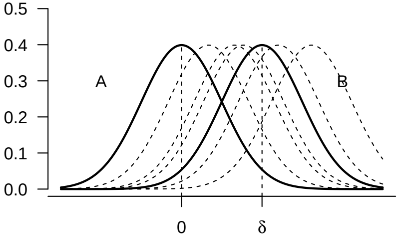


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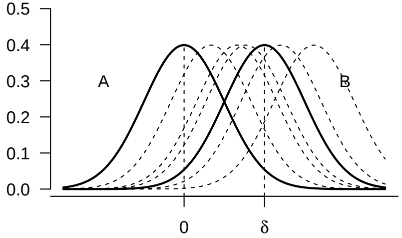


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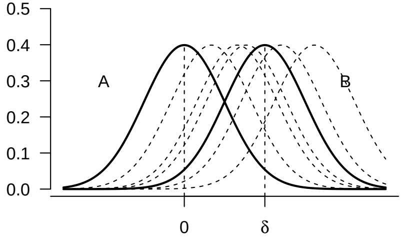


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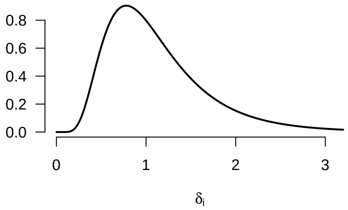
- Often more powerful tests of product differences
- Quantification of heterogeneity

# A DOD model for replications

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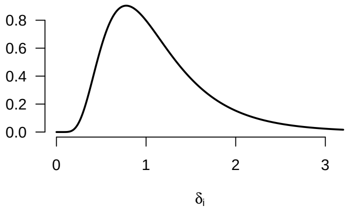


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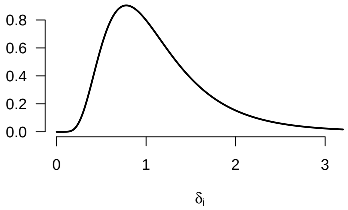


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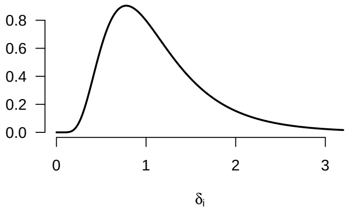


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Solution:

- Gauss-Hermite quadrature

# A replicated Degree of Difference example

25 panelists — 8 replications.

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| DOD test for reps | 5.03 | 1 | 0.0124 |
| DOD test prod+reps | 11.14 | 2 | 0.0038 |

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- How do these effects influence $d'$ and power?
- Are we able to distinguish between decision rules for DOD and A-not A with sureness?

Thanks to the scientific committee

Thank you for your attention!

# References

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