

From Many to Few: A Graph Theoretic Screening Tool for Product Developers

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“I saw an angel in the marble and carved until I set him free”

Background: Michelangelo reportedly carved his many masterpieces by removing all that was irrelevant to his final goal. While this approach clearly benefited the artist, it can also serve the brand and product developer in search of best combinations of items. Whether these items are juices in a mixed-juice drink box, flavor combinations for savory snacks or topping choices on a pizza, the practical problem is often the same, namely that from a moderate number of items an astounding number of combinations can be formed. While a range of techniques, from group discussion to fractional factorials and conjoint analysis, are currently used to trim down the full list of combinations to a list small enough for targeted testing, no technique in common use is specifically built to address this problem. Currently, much depends on the category-specific expertise of the product developer with the risk that surprising but potentially viable combinations might be mistakenly excluded from consideration. In this report we address this problem by recommending a new approach, based on relatively young mathematical techniques, that recognizes the special structure of this problem and allows us to systematically screen down a large list of combinations to one of manageable size.

Scenario: You work for a major pizza restaurant franchising corporation^a. Although your restaurants allow customers to order pizzas with any toppings they desire, there is also a franchise-wide menu of predetermined pizzas to help guide consumer choice and to allow restaurants to benefit from economies of scale. Based on input from marketing, upper management has determined that an updating of the current menu is warranted. The new menu is to be created based on a few standardizing assumptions for ease of rapid preparation. Specifically, each pizza will have a standard red sauce as its base and will have five or fewer toppings out of a pre-screened list of twenty-four available toppings in addition to a standard cheese topping. See Table 1. There are to be five pizzas altogether on the menu, and your team has been assigned the task of recommending potential menus to marketing for consideration.

Anchovy	Artichoke	Bacon	Basil
Broccoli	Chicken	Eggplant	Feta
Garlic	Green Pepper	Ham	Jalapeño
Mushroom	Olive	Onion	Pepperoni
Pineapple	Prosciutto	Red Onion	Red Pepper
Ricotta	Sausage	Spinach	Tomato

Table 1. Twenty-four possible pizza toppings.

As your team begins to consider various options it quickly becomes apparent that the number of potential pizzas is quite large. In fact, based on a simple calculation you determine that there are more than fifty-five thousand pizzas to choose from. Your team is able, based on

category-specific knowledge, to recommend many pizzas that appear to be of reasonable quality, but you would prefer a more systematic approach to guide your search. In particular, you would like to know that unusual combinations will receive at least some attention before being eliminated from future consideration. You realize that you need insight as to how to screen your list of fifty-five thousand pizzas down to a manageable number, such as twenty-five or thirty. Once you have a reduced list, you plan to apply traditional research tools, such as Total Unduplicated Reach and Frequency (TURF)^{1,2} to identify final recommendations that will span the consumer space as broadly as possible.

Trimming Combinations: The sheer number of options that arise when combinations are formed makes it very difficult to award each combination due attention. Therefore we recommend an indirect approach that quickly identifies combinations that can be safely eliminated from consideration. In particular, we use appropriateness³ as a metric and we hypothesize that a combination of toppings will be inappropriate whenever it is inappropriate to combine any of the items in the combination together as pairs^b. This hypothesis enables us to restrict attention to combinations that are fully pairwise appropriate. To identify these desirable combinations, we use tools from the mathematical field of graph theory.

A Graph Theoretic Screening Tool: Suppose for a moment that we knew all pairwise appropriateness information, so we could state which items were appropriate to combine in pairs. From this information we could draw a picture, called a graph, in which items appeared as nodes with edges drawn to indicate when it is appropriate to combine items⁵. See Figure 1.

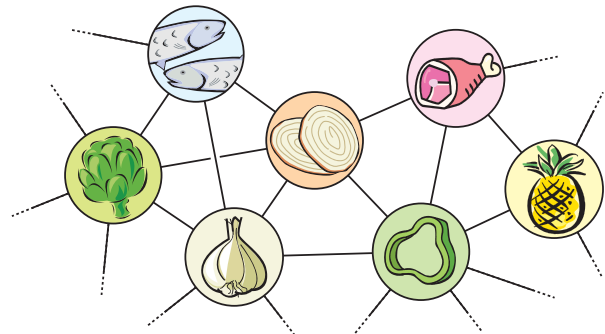


Figure 1. A graph showing pairwise appropriateness information for pizza toppings.

Such a graph can be analyzed to determine which collections of nodes are fully interconnected⁶. These sets of fully interconnected nodes are called cliques^c. See Figure 2. In the case of pizzas, cliques correspond to combinations of toppings that can all be appropriately combined in pairs. Perhaps more importantly, however, non-cliques correspond to combinations of toppings that contain at least one pair of toppings that are not appropriate to combine. See Figure 3, in which it is not appropriate to combine anchovies with ham, but it is appropriate to combine either with

onions. This means that by identifying and screening out non-cliques, we eliminate all combinations of toppings that contain at least one poorly matched pair. Depending on the level of appropriateness that we require, the list of cliques can be much smaller than the original list of combinations. Thus screening out the non-cliques gives us an extremely effective tool to trim the large set of all combinations down to a set of reasonable size⁸.

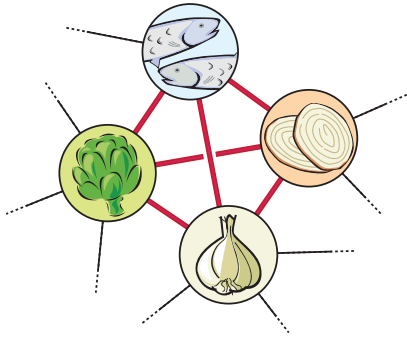


Figure 2. A clique of pizza toppings.

Screening Topping Combinations: In order to apply the above techniques, you design an online study in which 250 consumers are polled regarding the acceptability of the 276 possible pairs that can be formed from your full list of toppings. The pairs are presented to each consumer in a randomized and balanced fashion with the order of presentation within the pairs randomized as well. The consumers are informed of the context of the survey and for each pair are asked the single yes/no question, “Is it appropriate to combine these toppings on a pizza?” The consumers find these questions very easy to answer and, on average, complete the survey in less than forty minutes. For each pair, you determine the proportion of consumers that considered the combination of the items in that pair appropriate.

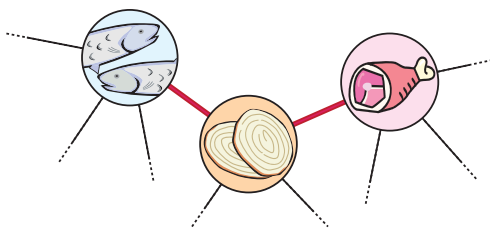


Figure 3. A non-clique of pizza toppings.

Your next step is to create a graph from your data and to separate cliques from non-cliques. For this step you must define what it means for pairs to be connected, and for this you allow your final goal to guide you. You start by defining as connected any pair of toppings that more than half of the consumers considered appropriate. This choice leads to many pairs being connected and to the formation of many large cliques. You increase your criterion for connectedness gradually, increasing the proportion of appropriateness judgements required to consider a pair appropriate until you find the smallest criterion that produces cliques of size five but none of size six. In this case the criterion for correctness that you determine requires that 68% of consumers find a given pair appropriate.

Your threshold found, you are left with a list of twenty-six pizzas with the desirable property that all toppings on each pizza are appropriate to combine in pairs. Conversely, all pizzas that contain even one pair of toppings that are not appropriate to combine have been eliminated. Thus, with 276 binary-response questions in an inexpensive study that was straightforward to conduct, you have screened more than fifty-five thousand possible pizzas down to a set small enough to be investigated using traditional tools. In particular, of your twenty-six candidates, three are quite different from combinations previously considered by your team. You conduct internal testing to confirm the viability of these novel combinations and then plan a follow-up study on the twenty-six pizzas using TURF analysis on which you will base your final recommendations.

Conclusion: By focusing attention on the elimination of combinations that contain inappropriate pairs we are able to screen a vast number of combinations down to a list small enough for traditional tools to be applied. Moreover, the data used to conduct this screening are straightforward to collect. Once such data have been collected, clique finding techniques from graph theory can be used to conduct the screening process. Since challenges involving combinations appear throughout product development, we expect newly developed graph theoretic techniques to appear increasingly often as chisels in the hands of brand and product developers as they free their angels from the previously impenetrable combinatorial marble.

Notes

- Scenario, including topping list, adapted from collaboration with Michael Nestrud.
- This hypothesis, called the principle of *supercombinatorality*, has been validated experimentally in a number of product categories⁴, including pizzas.
- Fully disconnected sets can also be informative⁷.

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