

Rotations in Product Tests and Surveys

Daniel M. Ennis, Benoît Rousseau, and John M. Ennis

Background: In this report we consider how to balance item positions, item sequences, and the spread of these sequences within rotations. The term “sequence” will be used to mean a pair of items occurring one after the other. The term “item” refers to a product, concept, or a question in a survey. Methods to account for sequential effects in product tests, or events and behaviors in survey research, play a role in product testing and survey design. Generally, interest centers on balancing the effect of one item on another to minimize bias and reduce test variance. There have been a number of approaches to addressing the design of tests so that sequential effects are accounted for and these include random presentation orders, the use of Latin Square designs, and computer searching through a large number of designs to meet a balancing criterion. In product testing over multiple days, which becomes necessary when large number of products are evaluated, sequences within days are much more important than between days and it is useful to consider how designs for this situation can be constructed.

Scenario: You are interested in evaluating blends of orange juice with other fruit juices among a random sample of 300 consumers from a target population. There are six samples to be evaluated and you plan to test them in sets of three over two days on a group of attributes, including liking. Since the taste of one blend may affect another, you expect sequential effects and would like to control for them. With 6 products, there are 30 unique sequences.

Position and Sequence Effects: It is a fairly straightforward task to balance products by position. For six items, only six different rotations are needed as shown in Table 1. Every item in Table 1 is evaluated once in every position and 50 replications of Table 1 would produce a position-balanced design for 300 consumers. However, Table 1 does not control for sequence effects and Table 2 shows an analysis of each sequence’s frequency when Table 1 is replicated 50 times. Of the 30 unique sequences, only 6 of them occur and each occurs 250 times.

Rotation	Position					
	DAY 1			DAY 2		
	1	2	3	4	5	6
1	A	B	C	D	E	F
2	B	C	D	E	F	A
3	C	D	E	F	A	B
4	D	E	F	A	B	C
5	E	F	A	B	C	D
6	F	A	B	C	D	E

Table 1. A basic set of rotations for six items that balances the items by position only.

First	Second					
	A	B	C	D	E	F
A		250	0	0	0	0
B	0		250	0	0	0
C	0	0		250	0	0
D	0	0	0		250	0
E	0	0	0	0		250
F	250	0	0	0	0	

Table 2. Counts of sequences for the rotations in Table 1.

Random Presentation Orders: One approach to creating a better sequence-balanced design is to randomize the items in each row. The problem with this approach is that the position balance may not be preserved as shown in Table 3 (counts range from 39-63) and sequences may not be balanced either as shown in Table 4 (counts range from 37-63). Although randomizing presentation orders is convenient, and often used in product testing, there are superior approaches.

Item	Position					
	1	2	3	4	5	6
A	52	55	47	55	39	52
B	39	49	56	50	51	55
C	49	51	49	44	55	52
D	52	48	48	40	63	49
E	63	54	52	47	42	42
F	45	43	48	64	50	50

Table 3. Counts by position and item for a random presentation order design.

First	Second					
	A	B	C	D	E	F
A		51	44	55	46	52
B	49		45	52	46	53
C	52	50		53	56	37
D	54	45	48		43	61
E	41	63	51	51		52
F	52	52	63	37	46	

Table 4. Counts of sequences for the random presentation order design of Table 3.

Williams Squares: An alternative to random presentation orders is to use Williams Squares¹, which are based on Latin Squares². An example of this design is shown in Table 5. Here position and sequences are balanced. Table 5 also shows, however, that the spread of the sequences across the positions is highly variable. If this design were repeated 50 times, sequence (A B) would always occur in the first and second positions and (C B) would always occur in the fifth and sixth positions. These two sequences would occur in these positions fifty times each and would appear nowhere else in the design. The replicated use of a Williams Square design such as Table 5, therefore, is not the most suitable design to balance the spread of sequence effects, although it is useful to balance for position and sequence effects, ignoring where they occur.

Rotation	Position					
	1	2	3	4	5	6
1	A	B	F	C	E	D
2	B	C	A	D	F	E
3	C	D	B	E	A	F
4	D	E	C	F	B	A
5	E	F	D	A	C	B
6	F	A	E	B	D	C

Table 5. A Williams Square design for six items balanced for position and sequences but not sequence spread.

Column Randomization: One way to create options is to search for designs to ensure that each item appears an equal number of times and that sequences and their spread

are balanced. In column randomization, each column is randomized as a unit and a design is selected from a large number of possibilities that creates the desired balance. If one begins with a design such as that in Table 1, where each item appears an equal number of times in each position, column randomization does not disturb position balance, but it does alter sequence balance. A measure of sequence balance is the variance of the counts for each sequence. When this number is zero, perfect sequence balance is achieved as occurs in a single Williams Square. Sets of sequence-balanced designs can then be tested for sequence spread and the best design chosen.

The Beverage Design: Beginning with 50 replicates of Table 1, you independently column randomized each table. In column randomization, the order of the entire column within the 6 x 6 table is randomized. Because of this, the position balance in Table 1 is preserved. The result is a design composed of 300 rows and 6 columns which can be analyzed for sequence balance. You calculate the number of occurrences of each of the 30 unique sequences and its variance. If the variance is zero, perfect sequence balance is achieved. You calculate sequence spread by determining the frequency for each of the 30 sequences in each of the five positions that they occupy [(1 2), (2 3), (3 4), (4 5), (5 6)]. If perfect sequence spread occurred in your beverage design, there would be 10 occurrences of each sequence in each of these positions and the variance of these counts would be zero.

Using a column randomization algorithm you search one million designs and retain only those designs with zero variance for sequence occurrences. There were 68 such designs and each one has perfect sequence balance. Then

Item	Position					
	1	2	3	4	5	6
A	50	50	50	50	50	50
B	50	50	50	50	50	50
C	50	50	50	50	50	50
D	50	50	50	50	50	50
E	50	50	50	50	50	50
F	50	50	50	50	50	50

Table 6. Counts for a column-randomized design by position and item.

First	Second					
	A	B	C	D	E	F
A		50	50	50	50	50
B	50		50	50	50	50
C	50	50		50	50	50
D	50	50	50		50	50
E	50	50	50	50		50
F	50	50	50	50	50	

Table 7. Counts of sequences for the column-randomized design of Table 6.

you search through these 68 designs for the one with the lowest sequence spread variance. Table 6 shows that your final design achieves perfect position balance and Table 7 shows that your sequence balance is also perfect, unlike Table 4 where random presentation orders were used. In the case of 6 products and 300 consumers, it is impossible to create a perfect spread of sequences across the design

because it would require 720 consumers to do so. However, it is possible to find designs that provide much better spread than others while achieving position and sequence balance. Table 8 shows the occurrences for (A B) and (C B) for each of the five positions. If spread was perfect, all of these numbers would be 10. The results are far superior to replicating a Williams Square where the counts would be either zero or 50. Figure 1 shows that for the design you chose, sequence spread is quite acceptable when all sequences are considered. This figure shows that the most common frequency is 10 with a range from 8 to 13.

Positions Orders	Times (A B) Occurs	Times (C B) Occurs
(1 2)	9	8
(2 3)	8	13
(3 4)	10	11
(4 5)	12	9
(5 6)	11	9

Table 8. The spread of the sequences (A B) and (C B) over testing positions for the column-randomized design.

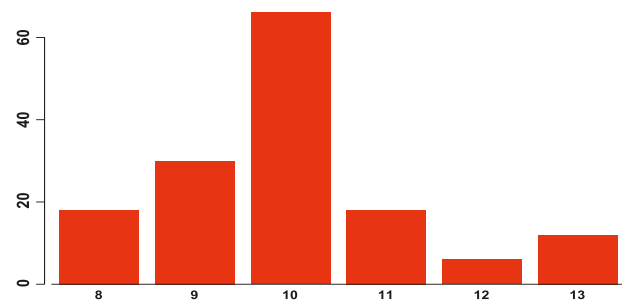


Figure 1. Number of times that the sequences occur in the five position orders and how they are clustered around 10.

Between Day Sequences: In a product test over several days, the sequential effect within a day is often much more important than between days. In Table 1, for instance, there may be a sequence effect from A → B and B → C, but little or no effect from C → D in rotation 1. Column randomization is well-suited to account for this difference by calculating the sequence variance only on the basis of the (1 2), (2 3), (4 5), and (5 6) position sequences. When there are a large number of products tested over multiple days, the benefit of this technique can become substantial.

Conclusion: Constructing test designs to account for position, sequence, and sequence spread in product tests and surveys (where the items may be questions or options within a question) can reduce bias and improve precision. There are a number of methods to construct these designs. The computer search method to create designs using column randomization may often be the most useful and flexible approach to finding optimal or close to optimal designs.

References and Notes

- Williams, E. J. (1949). Experimental designs balanced for the estimation of residual effects of treatments. *Australian Journal of Scientific Research*, Ser. A 2, 149-168.
- Cochran, W. G. and Cox, G. M. (1992). *Experimental Designs* (2nd ed.). New York, NY: John Wiley & Sons, Inc.