Multiplicative versus Ratio Comparisons

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Background: Claims such as "Our product whitens teeth twice as well as product X!" abound in advertising. Such claims have traditionally been viewed as statements regarding ratios of product performances and have been treated accordingly. In previous technical reports[1,2] we have both reviewed and extended traditional ratio based statistical methods used to evaluate such claims. In this report we offer a new perspective, one that is both more natural and more powerful. In particular, we propose a shift away from ratio based comparisons towards what we term multiplicative comparisons. This means that instead of viewing the above claim as a statement that our product's performance divided by the performance of product X is at least two, we instead consider whether the efficacy of our product is at least twice the efficacy of product X. This change might seem trivial, but we show below that this change is both more generally applicable and is more powerful than a ratio based approach. In what follows, we review our recent statistical work on a multiplicative approach to evaluate product claims and we compare this multiplicative approach to the ratio based approaches that follow in the classical tradition.

Scenario: Your company manufactures a carpet malodor treatment and your marketing department wants to make an advertising claim against one of your major competitors. You have been asked to determine what percentage can justifiably be used in a claim such as "Our product reduces malodor 10% more effectively than product X." As a pilot study you recruit 100 consumers and each consumer performs a pair of 2 alternative forced choice (2-AFC) trials. In each pair the consumers are presented with odor chambers containing either a malodorous sample or a malodorous sample plus treatment. Within each pair the consumers are presented with exactly one chamber of each type. The consumers are then asked which chamber in the pair was the most malodorous. Each consumer is tested on one pair involving your product and one pair involving your competitor's product. You balance the order of presentation and evaluation over the entire design. The results of your study are shown in Table 1.

Room	Correct	N	d' value	Variance of d'
Malodor + Your Product	64	100	0.51	0.033
Malodor + Competitor	52	100	0.07	0.031

Transforming the Data: In order to quantify the performance of your product relative to your competitor's you first need to transform your data to a scale on which such comparisons are meaningful. In particular you need to transform your data to measurements on a ratio scale. Following a Thurstonian approach you assume that each type of chamber in your study is represented by a distribution on an interval scale for which larger values indicate less malodor. Since there many sources of variance associated with the perception of malodor you assume that the perceptual distributions corresponding to each item are normally distributed. Let δ_1 be the difference between the mean associated with the malodor only and the mean associated with your product plus malodor, and let δ_2 be the difference between the mean associated with the malodor only and the mean associated with your competitor's product plus malodor, so that δ_1 and δ_2 are both positive. Assuming equal variance in these δ values, estimates of these δ values can be determined[3]. Since your examples involve large samples, you further assume that these estimates, called d_1' and d_2', are normally distributed. These d' values are positive differences of interval scale values relative to a common zero point, and hence have ratio scale properties. The d' values for your ratings data can be obtained using IFPrograms™ and are listed along with their variances in Table 1. Distributions of the d' values for your product, labeled "Product Y," and your competitor's product, labeled "Product X," are shown in Figure 1. Note that the d' value for your competitor's product has a nontrivial likelihood of being negative.

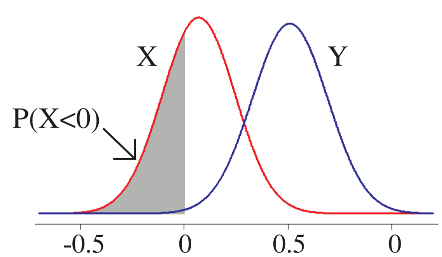

Table 1. Caption Body.

Ratio Based Analyses: In this example the ratio of the d' values is 7.28 = 0.51/0.07. Although it might seem that your product is seven times more effective than your competitor's, variation in these d' values means that if the experiment was run again the ratio could be very different. Thus you seek a lower 95% confidence bound on the ratio of the d' values. You start by consulting the classical statistical literature and find that variations on Fieller's method[4] are commonly used to determine such lower bounds. Unfortunately these classical methods require that your competitor's d' value be at least three times its standard deviation, meaning that Fieller's method is not applicable in your case. You then look to the modern extension of Fieller's method due to Ennis *et al*.[5] and find that although the extension is applicable no claim can be made. These results are clearly counter-intuitive given the qualitative superiority of your product.

A Multiplicative Perspective: The problem with using a ratio based approach in your application is that your competitor's product is very weak. This means that if you were to rerun your experiment there would be a reasonable chance that your competitor's product would have a deleterious effect. From a statistical standpoint this fact makes analysis of the ratio of the product performances problematic. In particular, when your competitor's product has close to no effect, the distribution of the ratio is very badly behaved. This is the reason that classical statistical methods place conditions on the denominator in the ratio. Although the recent extension of Ennis *et al*. allows for the consideration of arbitrary denominators, this extension is conservative as it does not penalize the denominator for its capacity to be negative.

An alternative perspective exists, however, which is to cease consideration of ratios and to begin consideration of multiplicative comparisons. Mathematically this means that instead of considering the expression $X/Y > c$ we consider the expression $X > cY$. This might seem like a trivial change but when there is a possibility that the denominator is negative it is not. This is because a statement such as "five is bigger than twice negative three" is true while the statement "five divided by negative three is bigger than two" is false. This new perspective allows for the computation of a lower confidence bound to compare product performances that will always be at least as large as the bound returned by a ratio based approach. The details of this new approach are laid out in Ennis and Ennis[6].

Determining a Lower Confidence Bound: Using the single integral expression in Ennis and Ennis you compute a lower confidence bound to compare the two products' performances and find that a claim of 8% better is possible in your case. In addition you determine that if the same performance levels were to hold in experiments with larger sample sizes that stronger claims would be possible. Although claims would become possible using the ratio based method as the sample size increased, these claims would not be as strong as those produced using the multiplicative approach. The results of this analysis are shown in Table 2.

Conclusion: Claims such as "Our product whitens teeth twice as well as product X" have traditionally been analyzed using statistics based on the ratios of the product performances. Although this approach is valid in many circumstances it is often conservative and is not always applicable. A shift in thinking from ratio based comparisons to multiplicative comparisons allows for confidence bounds to be found in a larger proportion of cases. In addition, confidence bounds found using the multiplicative based approach will always be at least as large as those found using the ratio based approach.

References:

[1]Ennis, D.M. (2006). Making Ratio Statements about Product Improvements. *IFPress*, **9**(*2*), 2-3.

[2]Ennis, J.M. and Ennis, D.M. (2008). Conditional Ratio Statements. *IFPress*, **11**(*2*), 2-3.

[3]Thurstone, L.L. (1927). A law of comparative judgment. *Psych. Review*, **34**, 268-389.

[4]Fieller, E.C. (1932). The distribution of the index in a normal bivariate population. *Biometrika*, **24**, 428-440.

[5]Ennis, D.M., Ennis, J.M., Palen, J., and Lampe, R. (2008). Confidence bounds for positive ratios of normal random variables. *Comm. in Stats.,* **37**, 207-317.

[6]Ennis, J.M. and Ennis, D.M. (submitted) Confidence Bounds for Multiplicative Comparisons. *Comm. in Stats.*, preprint available at www.ifpress.com.

Sample Size	Multiplicative		Ratio	
	Lower Bound	Percent Claim	Lower Bound	Percent Claim
100	1.081	8%	0.913	None
125	1.259	25%	1.096	9%
150	1.394	39%	1.236	23%
200	1.622	62%	1.474	47%
300	1.993	99%	1.863	86%
500	2.436	143%	2.333	133%