better approach is to use the variances in the *d′* values to estimate a lower bound on the ratio of product performance. You would then base your claim on the largest number for which repeated runs of the experiment yield a ratio of *d′* values at least as large as that number 95% of the time. Note that this probabilistic approach was advocated in our previous technical report on ratios and is mathematically equivalent under cerain conditions to the classical approach of Fieller in terms of estimating ratios[3,4,5,6].

Under many circumstances establishing a ratio claim in the manner described above would be straightforward. In your case however you have a problem because your competitor's product does not reduce malodor very well at all. Based on the results of your experiment it is conceivable that another run of the experiment would yield a negative value for the *d′* of your competitor's product. If this were to happen no positive ratio could accurately compare your product to your competitor's. In fact Figure 1 shows your competitor's product's poor performance could lead to a negative ratio of *d′* values more than 11% of the time. Ironically, without an additional statistical tool you might actually be penalized for having a weak competitor.

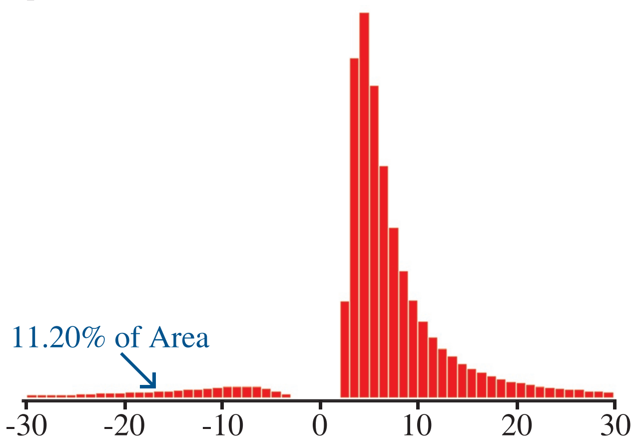

Figure 1. Results of Monte Carlo simulations showing the ratio of *d′*'s will be negative in 11% of the simulated runs of your experiment.

Conditional Ratio Statements: Recently Ennis et al.[7] extended the classical work on ratios to cases for which a competitor's weakness could produce a negative ratio. As a heuristic to understand this method you could again imagine running the original experiment over and over. This time however you would only consider runs of the experiment for which your competitor's product has a positive *d′*. Note that this approach is conservative since it allows your product to have either a positive or negative effect while only considering runs of the experiment for which your competitor has a positive average effect. Under these conditions you then find the largest number for which the ratio of *d′* values is at least as large as that number 95% of the time.

Results: Following this new method you find that the appropriate lower bound for your ratio is 3.04. This means that if you reran the original experiment until your competitor had a positive effect you would be 95% confident that the ratio of *d′* values would be greater than 3.04. This fact is illustrated in Figure 2. Based on this result you are now motivated to conduct a larger study to evaluate whether a claim that your product is 3 times more effective than your competitor's product at reducing malodor can be supported.

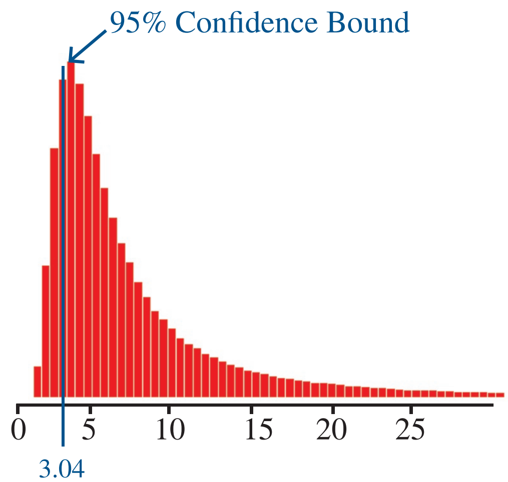

Figure 2. Results of Monte Carlo simulations showing 95% of the ratios of *d′*'s above 3.04 when simulated runs with negative competitor performance are excluded.

Conclusion: Once reliable interval scale data have been obtained, differences on interval scales can be used as terms in a ratio. Ratio claims can then be substantiated by a novel method that generalizes classical results such as Fieller's theorem and accommodates possible poor performance by a competitor.

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